Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4** **5**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For all integers x and y, if $3x + y^2 + 2$ is odd, then x is even or y is even.

You must begin by explicitly stating the contrapositive of the claim:

Solution: Let's prove the contrapositive. That is, for all integers x and y, if x is odd and y is odd, then $3x + y^2 + 2$ is even.

Let x and y be integers. Suppose that x and y are both odd. Then there are integers p and q such that $x = 2p + 1$ and $y = 2q + 1$.

Then

$$\begin{aligned} 3x + y^2 + 2 &= 3(2p+1) + (2q+1)^2 + 2 \\ &= (6p+3) + (4q^2 + 4q + 1) + 2 \\ &= 6p + 4q^2 + 4q + 6 \\ &= 2(3p + 2q^2 + 2q + 3) \end{aligned}$$

Let $t = 3p + 2q^2 + 2q + 3$. The above shows that $3x + y^2 + 2 = 2t$. Furthermore t must be an integer because p and q are integers. So $3x + y^2 + 2$ must be even.

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Discussion: Friday 11 12 1 2 3 4 5

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all rational numbers x, y and z, if y is non-zero, then $5(\frac{x}{y}) - 2z$ is rational.

Solution: Let x, y and z be rational numbers and suppose that y is non-zero.

By the definition of rational, $x = \frac{a}{b}$, $y = \frac{c}{d}$ and $z = \frac{e}{f}$, where the numbers a to f are all integers and b, d, and f are non-zero. Since y is non-zero, c must also be non-zero.

We can then compute

$$\begin{aligned} 5(\frac{x}{y}) - 2z &= 5(\frac{\frac{a}{b}}{\frac{c}{d}}) - 2\frac{e}{f} \\ &= 5(\frac{ad}{bc}) - 2\frac{e}{f} \\ &= \frac{5adf - 2ebc}{bcf} \end{aligned}$$

Let $p = 5adf - 2ebc$ and $q = bcf$. p and q are integers because a, b, c, d, e, and f are all integers. Futhermore, q is non-zero, because b, c, and f are all non-zero.

Therefore, $5(\frac{x}{y}) - 2z = \frac{p}{q}$, where p and q are integers and q is non-zero. So $5(\frac{x}{y}) - 2z$ is rational.