CS 173, Fall 18 Examlet 3, Part A

Name:____

NetID:____________________ **Lecture: B**

Discussion: Thursday Friday 11 12 1 2 3 4

$A = \{(a, b) \in \mathbb{R}^2 \ : \ a = 3 - b^2\}$

$B = \{(x, y) \in \mathbb{R}^2 \ : \ |x| \geq 1 \text{ or } |y| \geq 1\}$

Prove that A $\subseteq$ B. Hint: you may find proof by cases helpful.

Solution: Suppose that (a, b) is an element of A. Then, by the definition of A, $(a, b) \in \mathbb{R}^2$ and $a = 3 - b^2$.

Consider two cases, based on the magnitude of b:

Case 1: $|b| \geq 1$. Then (a, b) is an element of B. (Because it satisfies one of the two conditions in the OR.)

Case 2: $|b| < 1$. Then $b^2 < 1$. Then $a = 3 - b^2 > 3 - 1 = 2$. So $|a| \geq 1$, which means that (a, b) is an element of B.

So (a, b) is an element of B in both cases, which is what we needed to show.

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$A = \{(x, y) \in \mathbb{Z}^2 \mid 2xy + 6y - 5x - 15 \geq 0\}$

$B = \{(a, b) \in \mathbb{Z}^2 \mid a \geq 0\}$

$C = \{(p, q) \in \mathbb{Z}^2 \mid q \geq 0\}$

Prove that $(A \cap B) \subseteq C$.

Solution: Suppose that (x, y) is an element of $(A \cap B)$. This means that (x, y) is an element of A and (x, y) is an element of B. So $2xy + 6y - 5x - 15 \geq 0$ and $x \geq 0$, by the definitions of A and B.

Notice that $2xy + 6y - 5x - 15 = (x + 3)(2y - 5)$. So $(x + 3)(2y - 5) \geq 0$. We know that $x + 3$ is positive because $x \geq 0$. So we must have $(2y - 5) \geq 0$.

Now, if $(2y - 5) \geq 0$, then $2y \geq 5$. So $y \geq \frac{5}{2}$. So $y \geq 0$. This means that (x, y) is an element of C which is what we needed to show.