CS 173, Fall 18 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: B**

Discussion: Thursday Friday 11 12 1 2 3 4

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{3, 4\}$ $C = \{3, 7\}$

   $A \times (B \cap C) =$

   Solution: $A \times (B \cap C) = A \times \{3\} = \{(\text{fox}, 3), (\text{cat}, 3)\}$

   $A \cap B =$

   Solution: $A \cap B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap (B \cup C)$ $= (A \cap B) \cup (A \cap C)$ true for all sets A,B,C [√] false for all sets A,B,C [ ] true for some sets A,B,C [ ]

   $\forall x \in \mathbb{N}$, if $x < -10$, then $x = \pi$.
   (π is the familiar constant.) true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{12} + [9]^5$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5^2] = [25] = [3]$

   $[7]^8 = [3^2] = [9]$

   So $[7]^{12} = [7]^8 \cdot [7]^4 = [3] \cdot [9] = [27] = [5]$

   $[9]^2 = [81] = [4]$

   $[9]^4 = [4]^2 = [16] = 5$

   So $[9]^5 = [9] \cdot [5] = [45] = [1]$

   So $[7]^{12} + [9]^5 = [5] + [1] = [6]$.

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset$ is — an element of $\mathbb{Z}$ ☐ — a subset of $\mathbb{Z}$ ☑ — both ☐ — neither ☐

   $|A \cup B| = |A| + |B|$ — true for all sets A and B ☐ — true for some sets A and B ☑ — false for all sets A and B ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$