CS 173, Fall 18 Examlet 4, Part A

Name:____

NetID:________________________ **Lecture:** **B**

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) = (pq)(x + y)$

Prove that T is transitive.

Solution: Let (a, b), (p, q), and (m, n) be elements of A. Suppose that $(a, b)T(p, q)$ and $(p, q)T(m, n)$. By the definition of T, this means that $(xy)(p + q) = (pq)(x + y)$ and $(pq)(m + n) = (mn)(p + q)$

Since $m+n$ is positive, we can divide both sides by it, to get $(pq) = (mn)(p+q)/(m+n)$. Substituting this into the first equation, we get

$$(xy)(p + q) = (mn)(p + q)/(m + n) \times (x + y)$$

Multiplying both sides by $(m + n)$, we get

$$(xy)(p + q)(m + n) = (mn)(p + q)(x + y)$$

Since $(p + q)$ is positive, we can cancel it from both sides to get

$$(xy)(m + n) = (mn)(x + y)$$

By the definition of T, this means that $(a, b)T(m, n)$, which is what we needed to show.

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Suppose that T is a relation on the integers which is antisymmetric. Let's define a relation R on pairs of integers such that $(p,q)R(a,b)$ if and only if $(a+b)T(p+q)$ and bTq. Prove that R is antisymmetric.

Solution: Let (a,b) and (p,q) be pairs of integers. Suppose that $(a,b)R(p,q)$ and $(p,q)R(a,b)$.

By the definition of R, this means that $(a,b)R(p,q)$ means that $(p+q)T(a+b)$ and qTb. Similarly,$(p,q)R(a,b)$ means that $(a+b)T(p+q)$ and bTq.

Because T is antisymmetric, qTb and bTq implies that $q=b$. Similarly, $(p+q)T(a+b)$ and $(a+b)T(p+q)$ implies that $p+q=a+b$.

Since $q=b$ and $p+q=a+b$, $p=a$. So $(p,q)=(a,b)$, which is what we needed to prove.