CS 173, Fall 18 Examlet 5, Part A

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define the function $f : (0, \infty) \to \left(\frac{1}{3}, \infty\right)$ by $f(x) = \dfrac{x^2 + 2}{3x^2}$. Prove that f is onto.

   Solution: Let $y \in \left(\frac{1}{3}, \infty\right)$. Then $y > \frac{1}{3}$, so $3y > 1$, and therefore $3y - 1 > 0$.

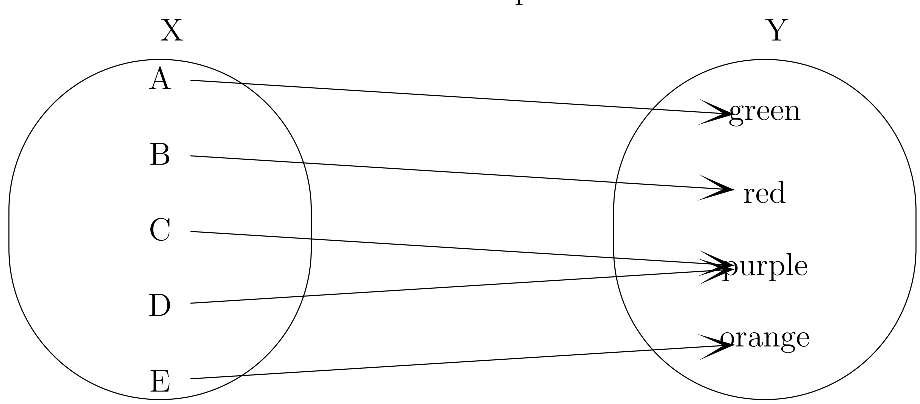
   So $\frac{2}{3y-1}$ is defined and positive. So consider $x = \sqrt{\frac{2}{3y-1}}$. x is defined and belongs to $(0, \infty)$.

   Then $x^2 = \frac{2}{3y-1}$. So $x^2 + 2 = \frac{2}{3y-1} + 2 = \frac{2+(6y-2)}{3y-1} = \frac{6y}{3y-1}$. And $3x^2 = \frac{6}{3y-1}$.

   Then $f(x) = \frac{x^2+2}{3x^2} = \frac{6y}{6} = y$.

   So we've found a pre-image for our original value y, which is what we needed to do.

2. (5 points) Complete this picture to make an example of a function that is onto but not one-to-one, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.



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1. (10 points) Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (h(x) - y, 3h(x) + 1)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

   Solution: Let (x, y) and (p, q) be elements of $\mathbb{Z}^2$ and suppose that $f(x, y) = f(p, q)$.

   By the definition of f, this means that $(h(x) - y, 3h(x) + 1) = (h(p) - q, 3h(p) + 1)$. So $h(x) - y = h(p) - q$ and $3h(x) + 1 = 3h(p) + 1$.

   Since $3h(x) + 1 = 3h(p) + 1$, $3h(x) = 3h(p)$. So $h(x) = h(p)$. Since h is one-to-one, this means that $x = p$.

   We now know that $h(x) = h(p)$ and $h(x) - y = h(p) - q$. Combining these equations, we get that $y = q$.

   Since $x = p$ and $y = q$, $(x, y) = (p, q)$, which is what we needed to prove.

2. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the co-domain and arrows showing how input values map to output values. The elements of the co-domain must be integers.