

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (10 points) If  $a$  is any real number,  $(a, \infty)$  is the set of all real numbers greater than  $a$ . Let's define the function  $f : (0, \infty) \rightarrow (\frac{1}{3}, \infty)$  by  $f(x) = \frac{x^2 + 2}{3x^2}$ . Prove that  $f$  is onto.

2. (5 points) Complete this picture to make an example of a function that is onto but not one-to-one, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.



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1. (10 points) Suppose that  $h : \mathbb{Z} \rightarrow \mathbb{Z}$  is one-to-one. Let's define  $f : \mathbb{Z}^2 \rightarrow \mathbb{Z}^2$  by  $f(x, y) = (h(x) - y, 3h(x) + 1)$ . Prove that  $f$  is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the co-domain and arrows showing how input values map to output values. The elements of the co-domain must be integers.

