

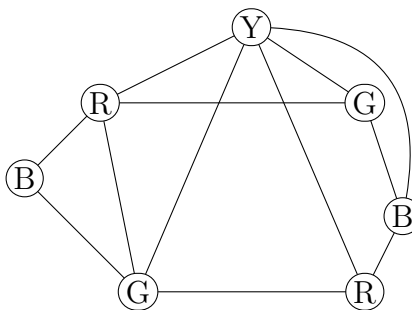
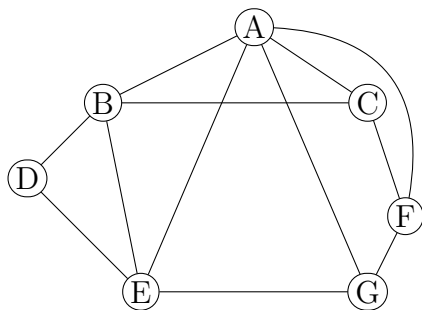
Name: \_\_\_\_\_

NetID: \_\_\_\_\_

Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



**Solution:** The chromatic number is four. The picture above shows how to color it with four colors (upper bound). For the lower bound, the graph contains a  $W_5$ : the hub is node A and the rim contains nodes B, C, F, G, and E.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a bipartite graph with at least one edge

1 ☐ 2 ☒ 3 ☐ can't tell ☐

Suppose I want to estimate  $\frac{103}{20}$ .  
3 is \_\_\_\_\_

an upper bound ☐ an exact answer ☐  
a lower bound ☒ not a bound on ☐

$\sum_{k=3}^n k^7$   $\sum_{p=1}^{n-2} p^9$  ☐  $\sum_{p=1}^{n-2} k^7$  ☐  $\sum_{p=1}^{n-2} k^9$  ☐  $\sum_{p=1}^{n-2} (p+2)^7$  ☒

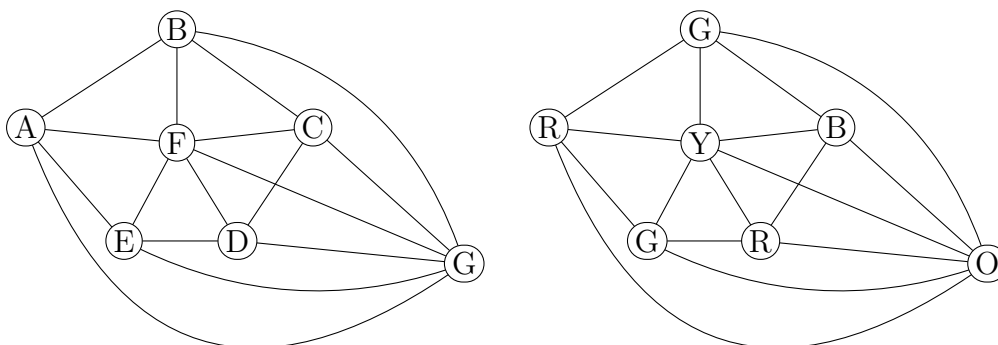
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Lecture: B

Discussion: Friday 11 12 1 2 3 4

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



**Solution:** The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a  $W_5$  whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a  $W_5$  requires four colors. Then the node G is connected to all six nodes in the  $W_5$ , so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of  $W_n$ .2 ☐3 ☐ $\leq 3$  ☐ $\leq 4$  ☒All elements of  $M$  are also elements of  $X$ . $M = X$  ☐ $M \subseteq X$  ☒ $X \subseteq M$  ☐

$$\sum_{k=0}^n \frac{1}{2^k}$$

$1 - \left(\frac{1}{2}\right)^{n-1}$  ☐

$2 - \left(\frac{1}{2}\right)^n$  ☒

$1 - \left(\frac{1}{2}\right)^n$  ☐

$2 - \left(\frac{1}{2}\right)^{n-1}$  ☐