CS 173, Fall 18 Examlet 8, Part B

Name: ____________________

NetID: ____________________ **Lecture:** B

Discussion: Friday 11 12 1 2 3 4

1. (8 points) Suppose we have a function g defined by

$$\begin{aligned} g(0) &= g(1) = c \\ g(n) &= kg(n-2) + n^2, \text{ for } n \geq 2 \end{aligned}$$

where k and c are constants. Express $g(n)$ in terms of $g(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

Solution:

$$\begin{aligned} g(n) &= kg(n-2) + n^2 \\ &= k(kg(n-4) + (n-2)^2) + n^2 \\ &= k(k(kg(n-6) + (n-4)^2) + (n-2)^2) + n^2 \\ &= k^3 g(n-6) + k^2(n-4)^2 + k(n-2)^2 + n^2 \end{aligned}$$

2. (2 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n!$. Give a recursive definition of f

Solution:

$f(0) = 1$, and $f(n) = nf(n-1)$ for $n \geq 1$.

You could also have used $f(n+1) = (n+1)f(n)$ for $n \geq 0$.

Name:____

NetID:____________________ **Lecture:** **B**

Discussion: **Friday** **11** **12** **1** **2** **3** **4**

1. (8 points) Suppose we have a function f defined by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n-1) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n-3)$ (where $n \geq 4$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

Solution:

$$\begin{aligned} f(n) &= 3f(n-1) + n^2 \\ &= 3(3f(n-2) + (n-1)^2) + n^2 \\ &= 3(3(3f(n-3) + (n-2)^2) + (n-1)^2) + n^2 \\ &= 27f(n-3) + 9(n-2)^2 + 3(n-1)^2 + n^2 \end{aligned}$$

2. (2 points) Suppose that G_0 is the graph consisting of a single vertex. Also suppose that the graph G_n consists of a copy of G_{n-1} plus an extra vertex v and edges joining v to each vertex in G_{n-1}. Give a clear picture or precise description of G_4.

Solution: This is a recursive construction of all the complete graphs, except for the indexing being off by one. So G_4 is just K_5.