CS 173, Fall 18 Examlet 9, Part A

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

(18 points) Here is a grammar G, with start symbol S and terminal symbols a and b.

$$\begin{aligned} S &\rightarrow a\ S\ a \mid S\ a\ S \mid a\ N\ a \\ N &\rightarrow a \mid b\ b \end{aligned}$$

Use (strong) induction to prove that any tree of height h matching (aka generated by) grammar G has at least h nodes with label a. Use $A(T)$ as shorthand for the number of a's in a tree T.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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(18 points) Recall that a node in a full binary tree must have either 0 or 2 children. A Sleepy tree is a full binary tree in which each node is colored orange or blue, such that:

- If v is a leaf node, then v may be colored orange or blue.
- If v has two children of the same color, then v is colored blue.
- If v has two children of different colors, then v is colored orange.

Use (strong) induction to show that the root of a Sleepy tree is blue if and only if the tree has an even number of orange leaves. You may assume basic divisibility facts e.g. the sum of two odd numbers is even.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: