Name:____

NetID:________________________ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Solution:

Proof by induction on n.

Base Case(s): At $n = 2$, $(2n)! = 4! = 24$. $2^n n! = 4 \cdot 2 = 8$. So $(2n)! > 2^n n!$

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $(2n)! > 2^n n!$ for all $n = 2, 3, \ldots, k$ for some integer $k \geq 2$.

Inductive Step: Notice that $2k + 1 \geq 1$ because k is positive. And $(2k)! > 2^k k!$ by the induction hypothesis.

So then

$(2(k+1))! = (2k+2)(2k+1)(2k)! \geq (2k+2)(2k)! > (2k+2)(2^k k!) = (k+1)2^{k+1}k! = 2^{k+1}(k+1)!$.

So $(2(k+1))! > 2^{k+1}(k+1)!$ which is what we needed to show.

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Discussion: Friday 11 12 1 2 3 4

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1+nx$.

Let x be a real number with $x > -1$.

Solution:

Proof by induction on n.

Base Case(s): At $n = 0$, $(1+x)^n = (1+x)^0 = 1$ and $1 + nx = 1 + 0 = 1$. So $(1+x)^n \geq 1+nx$.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $(1+x)^n \geq 1+nx$ for any natural number $n \leq k$, where k is a natural number.

Inductive Step: By the inductive hypothesis $(1+x)^k \geq 1+kx$. Notice that $(1+x)$ is positive since $x > -1$. So $(1+x)^{k+1} \geq (1+x)(1+kx)$.

But $(1+x)(1+kx) = 1 + x + kx + kx^2 = 1 + (1+k)x + kx^2$.

And $1 + (1+k)x + kx^2 \geq 1 + (1+k)x$ because kx^2 is non-negative.

So $(1+x)^{k+1} \geq (1+x)(1+kx) \geq 1 + (1+k)x$, and therefore $(1+x)^{k+1} \geq 1 + (1+k)x$, which is what we needed to show.