Name: _____

NetID: _____   **Lecture:**   B

Discussion: Friday   11   12   1   2   3   4

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $$T(4) = 7 \qquad T(n) = 2T\left(\tfrac{n}{4}\right) + d$$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $$2^n + 3^n \qquad n^3 \qquad 100\log n \qquad 3^{31} \qquad 3n\log(n^3) \qquad 7n! + 2 \qquad 173n - 173$$

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Discussion: **Friday   11   12   1   2   3   4**

1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$T(n) = T(n-1) + n$ $\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$ $\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$T(n) = 3T(n/3) + c$ $\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$\log_5 n$ is $\Theta(\log_3 n)$ ☐ $O(\log_3 n)$ ☐   neither of these ☐

Dividing a problem of size n into m sub-problems, each of size n/k, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐