CS 173, Fall 18 Examlet 12, Part A

Name:_____

NetID:_____ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

(8 points) Suppose that I have a set of p nodes, labelled 1 through p. How many different graphs can I make with this fixed set of nodes? (Isomorphic graphs with differently labelled nodes count as different for this problem.) Briefly justify your answer.

Solution: There are $\frac{p(p-1)}{2}$ possible edges for the graph. For each one, we can choose to include it or not. So there are $2^{\frac{p(p-1)}{2}}$ different possible graphs.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> For every computer game g, if g has trendy music or g has an interesting plotline, then g is not cheap.

Solution: There is a computer game g such that g has trendy music or an interesting plotline but g is cheap.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (zero or more of each variety).	$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☑
	$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐

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(9 points) Use proof by contradiction to show that $\log_5 2$ is irrational.

Solution: Suppose not. That is, suppose that $\log_5 2$ is rational. Then $\log_5 2 = \frac{a}{b}$, where a and b are integers, b non-zero.

Raising 5 to the power of both sides, we get $2 = 5^{\frac{a}{b}}$. Raising both sides to the bth power, we get $2^b = 5^a$. Since 2 and 5 are both prime, this equation can hold only if $a = b = 0$. But we know that b is non-zero. So we have a contradiction.

Since its negation led to a contradiction, our original claim must have been true.

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. At each timestep, she can make one move, which adds or deletes one plant from her arsenal. If she starts with 3 wallnuts, 2 peashooters, and 19 starfruit, how many different sequences of 25 moves will get her to a configuration with 7 wallnuts, 13 peashooters, and 9 starfruit?

Solution: The sequence of 25 moves needs to add 4 wallnuts, add 11 peashooters, and delete 10 starfruit. So we need to pick 4 moves in the sequence to be the ones that add wallnuts, and then 11 of the remaining 21 moves to be ones that add peashooters. So our number of choices is

$$\binom{25}{4}\binom{21}{11}$$