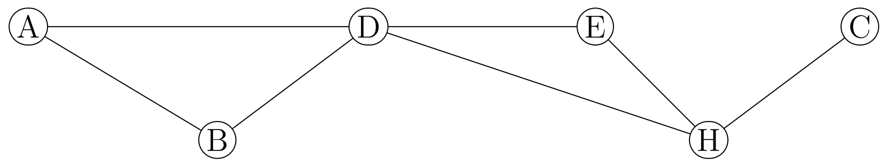
CS 173, Fall 18 Examlet 12, Part B

Name:____

NetID:________________________ **Lecture:** **B**

Discussion: **Friday 11 12 1 2 3 4**

Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3, 4\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, E) = n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

f(0) =

Solution: $\{E\}$

f(1) =

Solution: $\{D, H\}$

P =

Solution: $\{\emptyset, \{E\}, \{D, H\}, \{C, B, A\}\}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, P is not a partition of V. The subsets cover all of V with no partial overlap. However, P contains the empty set, since $f(3) = f(4) = \emptyset$.

(2 points) Check the (single) box that best characterizes each item.

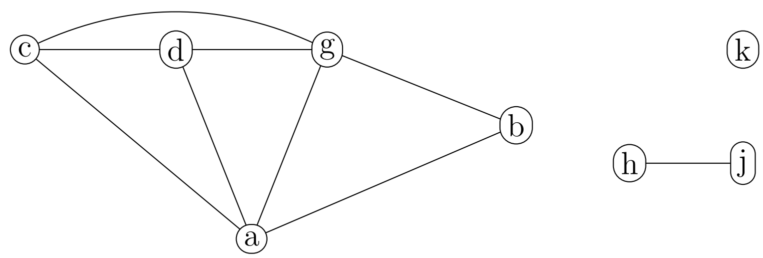
$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☑ sometimes ☐ never ☐

Name:____

NetID:______________________ **Lecture: B**

Discussion: Friday 11 12 1 2 3 4

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$ **Solution:** 8

$f(d) =$ **Solution:** $\{cd, ad, dg\}$

$f(h) =$ **Solution:** $\{hj\}$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, T is not a partition of E. T contains all edges in E. However, $f(k)$ is the empty set, so T contains the empty set. Also, there is partial overlap between the subsets, e.g. $f(d)$ and $f(a)$ are different but share the edge ad.

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

Solution: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$