Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Professor Martinez needs a state machine that will recognize the sequence 12120 when typed on a keypad. Specifically, it must read any sequence of the digits 0, 1, and 2. It should move into a final state immediately after seeing 12120, and then remain in that final state as further characters come in. For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

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Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(5 points) Let's say that two graphs are distinct if and only if they are not isomorphic. Is the set of distinct (finite) graphs countable or uncountable? Briefly justify your answer.

(10 points) Check the (single) box that best characterizes each item.

Every mathematical function $f : \mathbb{N} \to \mathbb{N}$ has a corresponding C++ program that will compute $f(n)$ given an input of n.	true ☐	false ☐	not known ☐
The rational numbers have the same cardinality as the integers.	true ☐	false ☐	not known ☐
If A is countably infinite, then is $\mathbb{P}(A)$ countably infinite?	always ☐	sometimes ☐	never ☐
The set of all (finite) binary trees where each node contains one of the letters A, B, or C	finite ☐	countably infinite ☐	uncountable ☐
The set of all intervals $[a, b]$ of the real line.	finite ☐	countably infinite ☐	uncountable ☐

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

(10 points) Check the (single) box that best characterizes each item.

Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false ☐

I found 143 identical marbles in my saucepan last Saturday. 143 is ____ how many marbles this size will fits in my saucepan.

an upper bound on ☐ exactly ☐
a lower bound on ☐ not a bound on ☐

$\forall x \in \mathbb{N}$, if $x < -10$, then $x = \pi$.
(π is the familiar constant.) true ☐ false ☐ undefined ☐

$f : \mathbb{N}^2 \to \mathbb{R}$
$f(p, q) = pq$ onto ☐ not onto ☐ not a function ☐

$|A - B| = |A| - |B|$

true for all sets A and B ☐ true for some sets A and B ☐
false for all sets A and B ☐

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(5 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n!$. Give a recursive definition of f

(10 points) Check the (single) box that best characterizes each item.

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

How many ways can I choose 6 bagels from among 8 varieties, if I can have any number of bagels from any type?

$\frac{8!}{6!2!}$ ☐ $\frac{13!}{6!7!}$ ☐ $\frac{14!}{9!5!}$ ☐

$\frac{14!}{6!7!}$ ☐ 8^6 ☐ 6^8 ☐

The running time of mergesort is recursively defined by $T(1) = d$ and $T(n) =$

$2T(n-1) + c$ ☐ $2T(n-1) + cn$ ☐

$2T(n/2) + c$ ☐ $2T(n/2) + cn$ ☐

The chromatic number of a full 3-ary tree

1 ☐ 2 ☐ ≤ 2 ☐

3 ☐ ≤ 3 ☐ can't tell ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

true ☐ false ☐