CS 173, Spring 18 Examlet 1, Part A

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur k, if k is blue, then k is not vegetarian or k is friendly.

Solution: There is a dinosaur k such that k is blue but k is vegetarian and k is not friendly.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every book b, if b is blue or b is not heavy, then b is not a math book.

Solution: For every book b, if b is a math book, then b is not blue and b is heavy.

3. (5 points) List all solutions to the equation $abc = 2$, where a, b, and c are integers. Notice that a solution where $a = 8$ and $b = 3$ would be different from a solution with $a = 3$ and $b = 8$.

Solution: **Solution:**

Writing the values for a, b, and c, in order, the possibilities are:

2, 1, 1	-2, -1, 1	-2, 1, -1	2, -1, -1
1, 2, 1	-1, -2, 1	-1, 2, -1	1, -2, -1
1, 1, 2	-1, -1, 2	-1, 1, -2	1, -1, -2

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1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$$(p \rightarrow q) \wedge (p \rightarrow \neg q) \equiv \neg p$$

p	q	$p \rightarrow q$	$p \rightarrow \neg q$	$(p \rightarrow q) \wedge (p \rightarrow \neg q)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tree t, if t grows in Canada, then t is not tall or t is a conifer.

Solution: For every tree t, if g is tall and t is not a conifer, then t doesn't grow in Canada.

3. (5 points) Suppose that m and p are positive integers such that $2p^2 + mp < 6$. What are the possible values for m? Briefly explain or show work.

Solution: Since $2p^2 + mp < 6$, $mp < 6 - 2p^2$. Since p is a positive integer $2p^2 \geq 2$. So $6 - 2p^2 \leq 4$. So $mp < 4$. Since m is a positive integer, this implies that m is 1, 2, or 3.

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1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p, q, r for which they produce different values.

$p \rightarrow (q \rightarrow r)$

$p \wedge (q \wedge r)$

Solution: Set p true, and set q and r false.

Then $(q \rightarrow r)$ is true because its hypothesis is false. Therefore $p \rightarrow (q \rightarrow r)$ is true.

However, $(q \wedge r)$ is false because both inputs are false. So $p \wedge (q \wedge r)$ is also false.

(It's sufficient to give one set of values that work. The general pattern is that p and/or q must be false.)

2. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> There is a relish r such that r is orange but r is not spicy.

Solution: For every relish r, r is not orange or r is spicy.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x + 7$ and $H(x) = \sqrt{x-1}$. Compute the value of $G(H(H(2)))$, showing your work.

Solution: $H(2) = \sqrt{1} = 1$

So $H(H(2)) = \sqrt{0} = 0$.

So $G(H(H(8))) = 0 + 7 = 7$

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1. (5 points) Are the following two expressions logically equivalent? Briefly justify your answer.

$$(p \wedge q) \rightarrow r \qquad\qquad (p \wedge \neg r) \rightarrow \neg q$$

Solution: These two expressions are equivalent. The first is false exactly when $(p \wedge q)$ is true and r is false. That is, it's false exactly when p and q are true and r is false.

The second is false exactly when $(p \wedge \neg r)$ is true and $\neg q$ is false. That is, it's false exactly when p is true, r is false, and q is true.

So the two expressions are false under exactly the same conditions and therefore they must be logically equivalent.

2. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every jedi j, if j has a light saber and j is not sick, then j can defeat the Dark Side.

Solution: There is a jedi j, such that j has a light saber and j is not sick, but j cannot defeat the Dark Side.

3. (5 points) Suppose that G and H are functions whose inputs and outputs are real numbers, defined by $G(x) = x^2$ and $H(x) = 2x - 10$. Compute the value of $G(H(G(3)))$, showing your work.

Solution: $G(3) = 9$

So $H(G(3)) = 2 \cdot 9 - 10 = 8$

So $G(H(G(3))) = 8^2 = 64$.

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

Solution: For every mushroom f, f is poisonous and f is not blue.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every alien A, if A has three fingers or A is not tall, then A is friendly.

Solution: For every alien A, if A is not friendly, then A does not have three fingers and A is tall.

3. (5 points) Find all integer solutions to $x^2 - 2x - 3 < 0$. Show your work.

Solution: Factoring the lefthand side, we get $(x + 1)(x - 3) < 0$. Since $x + 1$ is larger than $x - 3$, this means that $x + 1 > 0$ and $x - 3 < 0$. So $x > -1$ and $x < 3$. The only integers in this range are 0, 1, and 2.

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dog d, if d is a terrier, then d is not large and d is noisy.

Solution: There is a dog d, such that d is a terrier, but d is large or d is not noisy.

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dragon d, if d is green, then d is not large or d is fat.

Solution: For all dragons d, if d is large and d is not fat, then d is not green.

3. (5 points) Solve $5x + m = \frac{n}{5}$ for x, expressing your answer as a single fraction. Simplify your answer and show your work.

Solution:

$$\begin{aligned} 5x + m &= \frac{n}{5} \\ 5x &= \frac{n}{5} - m \\ x &= \frac{n}{25} - \frac{m}{5} = \frac{n - 5m}{25} \end{aligned}$$