CS 173, Spring 18 Examlet 2, Part A

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Prove the following claim, using direct proof and your best mathematical style.

For any integers m and k, if $k \leq 7$ and $0 < m - 3 \leq \frac{k}{7}$, then $m^2 - 9 \leq k$.

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(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a - b = nk$ for some integer n.

Claim: For all integers a, b, c, d, j and k (j and k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ and $j \mid k$, then $a + c \equiv b + d \pmod{j}$.

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(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definition of "divides." ($p \nmid q$ is the negation of $p \mid q$.)

For all integers k, a, b, if $k \nmid ab$, then $k \nmid a$ and $k \nmid b$.

You must begin by explicitly stating the contrapositive of the claim.

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(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers p and q ($p \neq -1$), if $\frac{2}{p+1}$ and $p + q$ are rational, then q is rational.

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(15 points) A natural number n is "snarky" if and only if $n = 3m + 1$, where m is a natural number. Use this definition and your best mathematical style to prove the following claim:

For all natural numbers x and y, if x and y are snarky, then $(x + y)^2$ is snarky.

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(15 points) A pair of positive integers (a, b) is defined to be a *partition* of a positive integer n if and only if $ab = n$. Using this definition and your best mathematical style, prove the following claim:

> For all positive integers a, b, and n, if (a, b) is a partition of n and $1 < a < \sqrt{n}$, then $\sqrt{n} < b < n$.