CS 173, Spring 18 Examlet 2, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Let a and b be integers, $b > 0$. The formula $a = bq + r$ partially defines the quotient q and the remainder r of a divided by b. What other constraint must we add to completely determine q and r?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2262, 546)$. Show your work.

   Solution:

   $2262 - 546 \times 4 = 2262 - 2184 = 78$

   $546 - 7 \times 78 = 0$

   So the GCD is 78.

3. (4 points) Check the (single) box that best characterizes each item.

   $7 \mid 0$ true ☑ false ☐

   $7 \equiv -7 \pmod{k}$ always ☐ sometimes ☑ never ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, bc) > 1$, then $\gcd(a, b) > 1$ or $\gcd(a, c) > 1$.

   Solution: This is true. If $\gcd(a, bc) > 1$, then there is some prime $p > 1$ that divides both a and bc. Since p is prime, p must divide a or b. So $\gcd(a, b) > 1$ or $\gcd(a, c) > 1$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2079, 759)$. Show your work.

   Solution:

   $2079 - 759 \times 2 = 2079 - 1518 = 561$

   $759 - 561 = 198$

   $561 - 198 \times 2 = 561 - 396 = 165$

   $198 - 165 = 33$

   $165 - 33 \times 5 = 0$

   So the GCD is 33.

3. (4 points) Check the (single) box that best characterizes each item.

   $-2 \equiv 2 \pmod 9$ true ☐ false ☑ (√)

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$ true ☑ (√) false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all natural numbers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is true. If $a \mid b$ and $b \mid a$, then $b = pa$ and $a = qb$, where p and q are integers. Since a and b are natural numbers and therefore not negative, p and q cannot be negative. So $b = pqb$, so $pq = 1$. So $p = q = 1$ and therefore $a = b$. [This is more detail than you'd need for full credit.]

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2385, 636)$. Show your work.

   Solution:

   $2385 - 3 \times 636 = 2385 - 1908 = 477$

   $636 - 477 = 159$

   $477 - 3 \times 159 = 0$

   So the GCD is 159.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod 5$ true [√] false [ ]

   For any integers p and q, if $p \mid q$ then $p \leq q$. true [ ] false [√]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $p \mid q$, then $s \equiv t \pmod{q}$.

   Solution: This is false.

   Informally, since q is larger than p, congruence mod q makes finer distinctions among numbers than p does.

   More formally, consider $s = 1$, $t = 4$, $p = 3$ and $q = 6$. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(221, 1224)$. Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \mid 0$ true [√] false [ ]

   $k \equiv -k \pmod{7}$ always [ ] sometimes [√] never [ ]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid c$ and $b \mid c$, then $ab \mid c$

   Solution: This is not true. Consider $a = b = c = 5$. Then $a \mid c$ and $b \mid c$. But $ab = 25$ and $c = 5$. So ab does not divide c.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7839, 1474)$. Show your work.

   Solution:

   $7839 - 5 \times 1474 = 7839 - 7370 = 469$

   $1474 - 3 \times 469 = 1474 - 1407 = 67$

   $469 - 7 \times 67 = 0$

   So the GCD is 67.

3. (4 points) Check the (single) box that best characterizes each item.

$-11 \equiv 4 \pmod 7$	true ☐	false ☑
For any positive integers p, q, and k, if $p \equiv q \pmod k$, then $p^2 \equiv q^2 \pmod k$	true ☑	false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod{p}$ and $q \mid p$, then $s \equiv t \pmod{q}$.

   Solution: This is true.

   Informally, since q is smaller, congruence mod q makes coarser distinctions than congruence mod q. So this is in the right direction and the relationship $q \mid p$ ensures that the details work out.

   More formally, from $s \equiv t \pmod{p}$ and $q \mid p$, we get that $s = t + pk$ and $p = qj$, where k and j are integers. So $s = t + q(jk)$, which means that $s \equiv t \pmod{q}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(4340, 1155)$. Show your work.

   Solution:

   $4340 - 1155 \times 3 = 4340 - 3465 = 875$

   $1155 - 875 = 280$

   $875 - 280 \times 3 = 875 - 840 = 35$

   $280 - 35 \times 24 = 0$

   So the GCD is 35.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod{5}$ true [√] false [ ]

   $\gcd(k, 0)$ for k positive 0 [ ] k [√] undefined [ ]