CS 173, Spring 18 Examlet 3, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

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For any integers s and t define $L(s,t)$ as follows:

$$L(s,t) = \{sx + ty \mid x, y \in \mathbb{Z}\}$$

Thus, $L(s,t)$ consists of all integers that can be expressed as the sum of multiples of s and t. Prove the following claim using your best mathematical style and the following definition of congruence mod k: $p \equiv q \pmod{k}$ if and only if $p = q + kn$ for some integer n.

Claim: For any integers a, b, r, where r is positive, if $a \equiv b \pmod{r}$, then $L(a,b) \subseteq L(r,b)$.

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$A = \{(p, q) \in \mathbb{R}^2 \ | \ p = 0\}$

$B = \{(x, y) \in \mathbb{R}^2 \ | \ (x-1)^2 + y^2 = 4\}$

$C = \{(s, t) \in \mathbb{R}^2 \ | \ (s+1)^2 + t^2 = 4\}$

Prove that $B \cap C \subseteq$ A.

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$A = \{(x, y, z) \in \mathbb{Z}^3 \ : \ 2x + y = z - 1\}$

$B = \{(a, b, c) \in \mathbb{Z}^3 \ : \ 2a - b = c - 3\}$

$C = \{(p, q, r) \in \mathbb{Z}^3 \ : \ r \text{ is even}\}$

Prove that $A \cap B \subseteq C$. (Work directly from the definition of "even.")

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$A = \{(a, b) \in \mathbb{R}^2 \ : \ a = 3 - b^2\}$

$B = \{(x, y) \in \mathbb{R}^2 \ : \ |x| \geq 1 \text{ or } |y| \geq 1\}$

Prove that A $\subseteq$ B. Hint: you may find proof by cases helpful.

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$A = \{\lambda(3,2) + (1-\lambda)(5,0) \mid \lambda \in [0,1]\}$

$B = \{(x,y) \in \mathbb{R}^2 \mid x \geq y\}$

Prove that $A \subseteq B$.

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$A = \{(x, y, z) \in \mathbb{Z}^3 \ : \ 0 < x \leq y \leq z\}$

$B = \{(a, b, c) \in \mathbb{R}^3 \ : \ c^2 \leq a\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \ : \ p \geq 1\}$

Prove that $A \cap B \subseteq C$.