CS 173, Spring 18 Examlet 3, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{fox}, \text{cat}\}$ $B = \{\text{rat}, \text{mouse}\}$

   $A \cap B =$

   Solution: $\emptyset$

   $\{p^2 + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 2 \text{ and } 1 \leq q \leq 3\} =$

   Solution: $\{2, 3, 4, 5, 6, 7\}$

2. (4 points) Check the (single) box that best characterizes each item.

   For all integers n, if $n^2 = 101$, then $n > 11$. true [√] false [ ] undefined [ ]

   If $x \in A \cup B$, then $x \in A$. true for all sets A and B [ ] true for some sets A and B [√] false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^6 + [5]^3$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^6 = [3]^3 = [27] = [5]$

   $[5]^3 = [125] = [4]$

   $[6]^6 + [5]^3 = [5] + [4] = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \times C \subseteq B \times C$.

   Solution: This is true. Suppose we pick (x, y) from $A \times C$. Then x is in A and y is in C. Since x is in A and $A \subseteq B$, x is in B. So (x, y) is in $B \times C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset$ is an element of $\mathbb{Z}$ ☐ both ☐ a subset of $\mathbb{Z}$ ☑ neither ☐

   Sets A and B are disjoint $A \cap B = \{\emptyset\}$ ☐ $A \cap B = \emptyset$ ☑ $|A \cap B| = 1$ ☐ $A = \overline{B}$ ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{22} = [8]^{16} \cdot [8]^4 \cdot [8]^2 = [3][4][9]$

   $[3][4][9] = [3][36] = [3][3] = [9]$

   So $[8]^{22} = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \cup (B - C) \subseteq (A \cup B) - C$

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{3, 4\}$, and $C = \{2, 3\}$.
   Then $A \cup (B - C) = \{1, 2\} \cup \{4\} = \{1, 2, 4\}$.
   But $(A \cup B) - C = \{1, 2, 3, 4\} - \{2, 3\} = \{1, 4\}$.
   So $A \cup (B - C) \not\subseteq (A \cup B) - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = A$

   true for all sets A and B ☐ false for all sets A and B ☐
   true for some sets A and B ☑

   $\emptyset \times \emptyset =$

   $\emptyset$ ☑ $\{\emptyset\}$ ☐ $\{\emptyset, \emptyset\}$ ☐ $\{(\emptyset, \emptyset)\}$ ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[6]^8 + [5]^{20}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:
   $[6]^2 = [36] = [10]$
   $[6]^4 = [10]^2 = [100] = [9]$
   $[6]^8 = [9]^2 = [81] = [3]$
   $[5]^2 = [25] = [-1]$
   $[5]^4 = [1]$. So $[5]^{16} = [1]$. So $[5]^{20} = [5]^{16} \times [5]^4 = [1][1] = [1]$.
   $[6]^8 + [5]^{20} = [3] + [1] = [4]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

   Solution: This is true. An element of $A \cap C$ must be in both A and C. I $A \subseteq B$, then it's also in B. But then it's in $B \cap C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A = \overline{A}$ (Assume the universe is not empty.)

   true for all sets A ☐ true for some sets A ☐

   false for all sets A ☑

   $\forall x \in \mathbb{Q}$, if $x^2 = 3$, then $x > 1000$.

   true ☑ false ☐ undefined ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{40}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{40} = [7]^{32} \cdot [7]^8 = [5] \cdot [-2] = [-10] = [1]$

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1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{oak}, \emptyset\}$

   $A \cap B =$

   Solution: $\{\text{oak}\}$

   $\{(p, q) \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} =$

   Solution: $\{(p, q) \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} = \{(1, 6), (6, 1), (2, 3), (3, 2)\}$

2. (4 points) Check the (single) box that best characterizes each item.

$\emptyset \in A$	true for all sets A	☐	true for some sets A	☑
	false for all sets A	☐		
$\lvert A \cup B\rvert = \lvert A\rvert + \lvert B\rvert$	true for all sets A and B	☐	true for some sets A and B	☑
	false for all sets A and B	☐		

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{32} = [3]^2 = [9]$

   $[8]^{37} = [8]^{32} \cdot [8]^4 \cdot [8] = [9] \cdot [4] \cdot [8] = [36] \cdot [8] = [3] \cdot [8] = [24] = [2]$

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1. (4 points) $A = \{\text{fox, tiger, wolf}\}$ $B = \{3, 4\}$ $C = \{6, 7, 8\}$

   $| A \times (B \cup C) | =$

   Solution: $|\text{fox, tiger, wolf} \times \{3, 4, 6, 7, 9\}| = 3 \times 5 = 15$

   $\{p + q \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ 1 \leq p \leq 3 \text{ and } 1 \leq q \leq 3\} =$

   Solution: $\{2, 3, 4, 5, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\{1, 2\} \cup \emptyset =$

$\emptyset$		$\{\emptyset\}$		$\{1, 2\}$	√
$\{(1, \emptyset), (2, \emptyset)\}$		$\{1, 2, \emptyset\}$		undefined	

   $A \cup B = A$

true for all sets A and B		false for all sets A and B	
true for some sets A and B	√		

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[10]^{43} + [7]^{10}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[10] = [-1]$. So $[10]^{43} = [-1]^{43} = -1$.

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5]^2 = [25] = [3]$

   $[7]^8 = [3]^2 = [9]$

   So $[7]^{10} = [7]^2 \times [7]^8 = [5][9] = [45] = [1]$

   So $[10]^{43} + [7]^{10} = [1] + [-1] = [0]$.