CS 173, Spring 18 Examlet 4, Part A

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) < (pq)(x + y)$

Prove that T is transitive.

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Suppose that n is some integer ≥ 2. Let's define the relation R_n on the integers such that aR_nb if and only if $a \equiv b + 1 \pmod{n}$. Prove the following claim

Claim: If R_n is symmetric, then $n = 2$.

You must work directly from the definition of congruence mod k, using the following version of the definition: $x \equiv y \pmod{k}$ iff $x - y = mk$ for some integer m. You may use the following fact about divisibility: for any non-zero integers p and q, if $p \mid q$, then $|p| \leq |q|$.

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Let $A = \{(x, y) \in \mathbb{R}^2 \mid x + y = 10\}$. Consider the relation T on A defined by

$(a, b)T(p, q)$ if and only if $aq \geq bp$

Prove that T is antisymmetric.

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Let T be the relation defined on $\mathbb{N}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or $(x = p$ and $y \leq q)$

Prove that T is transitive.

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Let's define a relation T between natural numbers follows:

aTb if and only if $a = b + 2k$, where k is a natural number

Working directly from this definition, prove that T is antisymmetric.

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Suppose that T is a relation on the integers which is antisymmetric. Let's define a relation R on pairs of integers such that $(p, q)R(a, b)$ if and only if $(a + b)T(p + q)$ and bTq. Prove that R is antisymmetric.