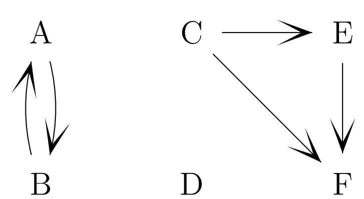
Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Recall that $\mathbb{N}^2$ is the set of all pairs of natural numbers. Let's define the equivalence relation $\sim$ on $\mathbb{N}^2$ as follows: $(x, y) \sim (p, q)$ if and only $|x - y| = |p - q|$. List three members of $[(2, 3)]$.

   Solution: (2,3), (3,4), (14,13)

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R an equivalence relation?

   Solution: Yes, R is an equivalence relation. It's reflexive because elements are always related to themselves. Since there aren't any relations between distinct elements, it's also symmetric and transitive.

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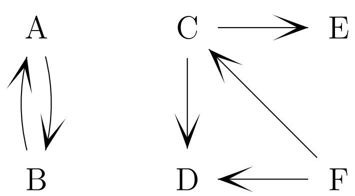
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Can a relation be symetric and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. Consider the relation R on the integers such that aRb if and only if $a = b$. This is both symmetric and antisymmetric.

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "disjoint" relation D on J by $(a, b)D(c, d)$ if and only if $b \leq c$ or $d \leq a$. Is D transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: D is not transitive. Consider $(1, 2)$, $(3, 5)$, and $(4, 6)$. Then $(4, 6)D(1, 2)$. $(1, 2)D(3, 5)$ and But not $(4, 6)D(3, 5)$.

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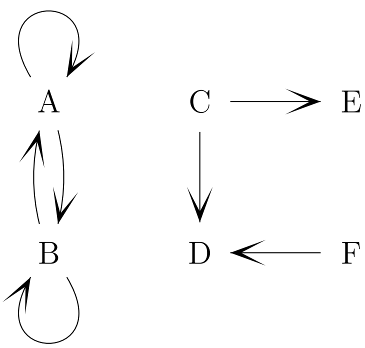
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☑ $\surd$

2. (5 points) Suppose that R is a partial order on a set A. What additional property is required for R to be a linear order (aka total order)? Give specific details of the property, not just its name.

   Solution: All pairs of elements must be comparable. That is, for any elements x and y in A, either xRy or yRx.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $2 \mid (x + y + 1)$. Is R transitive?

   Solution: No, R is not transitive. For example, $2R3$ and $3R4$ but it's not the case that $2R4$.

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NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

A C E

B D F

(that is, 6 nodes and no arrows at all)

Reflexive:	☐	Irreflexive:	√
Symmetric:	√	Antisymmetric:	√
Transitive:	√		

2. (5 points) Let R be the equivalence relation on the real numbers such that xRy if and only if $\lfloor x \rfloor = \lfloor y \rfloor$. Give three members of the equivalence class [13].

   Solution: 13, 13.1, 13.7

3. (5 points) Suppose that R is a relation on pairs of integers such that $(x, y)R(a, b)$ if and only if $x - a \geq 2$ and $y \geq b$. Is R a partial order?

   Solution: No, R is not a partial order. It's transitive and antisymmetric. However it's impossible for an element (x, y) to be related to itself, because that would require $x - x \geq 2$. So it's not reflexive.

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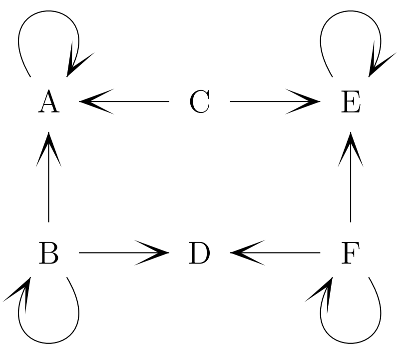
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ $\surd$

Transitive: ☑ $\surd$

2. (5 points) **Notice that this problem was corrected early in the exam. This is the corrected version.** Let's define the relation $\sim$ on $\mathbb{Z}$ sucn that $x \sim y$ if and only $|x - y| = 3$. List all elements related to 7.

   Solution: 4 and 10

3. (5 points) Let S be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)S(p, q)$ if and only if $x^2 + y^2 \leq p^2 + q^2$. Is S antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not antisymmetric. We have $(0, 1)S(1, 0)$ and $(1, 0)S(0, 1)$, but $(0, 1) \neq (1, 0)$.

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Name:____

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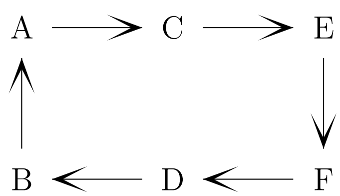
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☑

Transitive: ☐

2. (5 points) Suppose that R is an equivalence relation on a set A. Using precise set notation, define $[x]_R$, i.e. the equivalence class of x under the relation R.

Solution: $[x]_R = \{y \in A \mid xRy\}$

3. (5 points) Let J be the set of open intervals of the real line, i.e $J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$. Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Solution: This relation is not transitive. Consider $(1, 2)$, $(2, 3)$, and $(3, 4)$. Then $(1, 2)T(2, 3)$ and $(2, 3)T(3, 4)$, but not $(1, 2)T(3, 4)$.