CS 173, Spring 18 Examlet 5 (only one sheet)

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 13-letter strings can be made be rearranging the letters in the word ‘‘`massachusetts`’’? Show your work.

   Solution: There are 13 letters total, with 4 copies of s, two t's, and 2 a's. So the number of possibilities is

   $$\frac{13!}{4!2!2!}$$

2. (12 points) Check the (single) box that best characterizes each item.

   If a function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing, it must be one-to-one. true [√] false [ ]

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ onto [√] not onto [ ] not a function [ ]

   $g : (0, \frac{\pi}{2}) \to \mathbb{R}$, $g(x) = \sin(x)$ one-to-one [√] not one-to-one [ ] not a function [ ]

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, every color appears on at least two mailboxes. true [ ] false [√]

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x = xy$ true [√] false [ ]

   $(f \circ g)(x)$ $f(g(x))$ [√] $g(f(x))$ [ ] neither [ ]

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from $A \times B$ to C?

   Solution: There are pq elements in $A \times B$. So there are $(n)^{pq}$ ways to build a function from $A \times B$ to C.

2. (12 points) Check the (single) box that best characterizes each item.

   If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false ☑

   $g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$ one-to-one ☑ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = \lfloor x \rfloor$ onto ☑ not onto ☐ not a function ☐

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. Each slogan appears on at least two shirts. true ☐ false ☑

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x \neq y$ and $x + y = 0$ true ☐ false ☑

   Suppose $f : A \to B$. For all $x, y \in A$, if $f(x) = f(y)$, then $x = y$. onto ☐ one-to-one ☑ neither ☐

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1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can we form as many as possible into pairs, where each pair consists of one man and one woman?

   Solution: Since we're going to run out of men first, we need to construct a one-to-one function from the men to the women. Since we have 10 input elements and 15 output elements, the number of different functions is

   $$\frac{15!}{5!}$$

2. (12 points) Check the (single) box that best characterizes each item.

   If $f : A \to B$ is onto, then $|A| \geq |B|$ [√] $|A| \leq |B|$ [ ] $|A| = |B|$ [ ]

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 21$ (x odd)
   onto [√] not onto [ ] not a function [ ]

   $g : \mathbb{N} \to \mathbb{Z}$
   $g(x) = x^2$
   one-to-one [√] not one-to-one [ ] not a function [ ]

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. At least three of these shirts must have the same slogan.
   true [√] false [ ]

   $\forall x \in \mathbb{R},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$
   true [ ] false [√]

   Suppose $f : A \to B$. For all $x \in A$, there is a $y \in B$, $f(x) = y$.
   onto [ ] one-to-one [ ] neither [√]

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1. (5 points) How many different 9-letter strings can be made be rearranging the letters in the word ‘‘`silliness`’’? Show your work.

   Solution: There are 9 letters total, with 3 copies of s, two l's, and 2 i's. So the number of possibilities is

   $$\frac{9!}{3!2!2!}$$

2. (12 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the co-domain has exactly one pre-image. true ☐ false ☑

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 4$ (x even),
   $f(x) = x - 21$ (x odd)
   one-to-one ☐ not one-to-one ☑ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{R}$
   $f(p, q) = pq$
   onto ☐ not onto ☑ not a function ☐

   Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. There is a slogan that appears on at least two shirts. true ☑ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$ true ☐ false ☑

   Suppose $f : A \to B$. For all $x, y \in A$, if $x = y$, then $f(x) = f(y)$. onto ☐ one-to-one ☐ neither ☑

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1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, shocking pink, and neon green. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

   Solution: She needs to pull out five socks. By the pigeonhole principle, five socks and only four colors means that two must have the same color.

2. (12 points) Check the (single) box that best characterizes each item.

   The composition of two one-to-one functions is one-to-one. true [√] false [ ]

   $g : \mathbb{R} \to \mathbb{R}$, $g(x) = \sin(x)$ one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{R}$, $g(x) = \lfloor x \rfloor$ onto [ ] not onto [√] not a function [ ]

   If $f : \mathbb{N} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. domain [√] co-domain [ ] image [ ] none of these [ ]

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$ true [√] false [ ]

   Suppose $f : \mathbb{R} \to \mathbb{R}$. For all $x, y \in \mathbb{R}$, if $x < y$, then $f(x) < f(y)$. increasing [ ] strictly increasing [√] neither [ ]

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 2$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: It doesn't matter what the elements of A and B are, so let's suppose that $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Two elements of A must map to the same output value, with the third element x mapping to the other output value. There are three choices for which element x is. And then there are two choices for which output value corresponds to x. So 6 onto functions total.

2. (12 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has at least one pre-image. true [√] false [ ]

   $g : \mathbb{R} \to \mathbb{Z}$
   $g(x) = \lfloor x \rfloor$ one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{R}$
   $g(x) = x - 0.314$ onto [ ] not onto [√] not a function [ ]

   Each dorm room is given an integer access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then there is some access code that is shared by at least two rooms. true [√] false [ ]

   $\forall m, n \in \mathbb{Z},\ \exists x \in \mathbb{Q},\ x = \frac{m}{n}$ true [ ] false [√]

   Suppose $f : A \to B$. For all $y \in B$, there is an $x \in A$, $f(x) = y$. onto [√] one-to-one [ ] neither [ ]