

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that $g : \mathbb{N} \rightarrow \mathbb{N}$ is one-to-one. Let's define the function $f : \mathbb{N}^2 \rightarrow \mathbb{N}^2$ by the equation $f(x, y) = (x + g(y), g(x))$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that $f : \mathbb{Z} \rightarrow \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \rightarrow \mathbb{Z}^2$ by $g(x, y) = (f(x) + y, y + 3)$. Prove that g is onto.

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that $f : (0, \infty) \rightarrow (\frac{5}{4}, \infty)$ is defined by $f(x) = \frac{5x^2+3}{4x^2}$. Proof that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that $f : \mathbb{Z}^2 \rightarrow \mathbb{Z}$ is defined by $f(x, y) = xy + yx^2 - x^2$. Prove that f is onto.

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that A and B are sets. Suppose that $f : B \rightarrow A$ and $g : A \rightarrow B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that g is one-to-one.

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose that $f : [0, \frac{1}{2}] \rightarrow [1, \frac{5}{2}]$ is defined by $f(x) = \frac{x^2+1}{1-2x^2}$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.