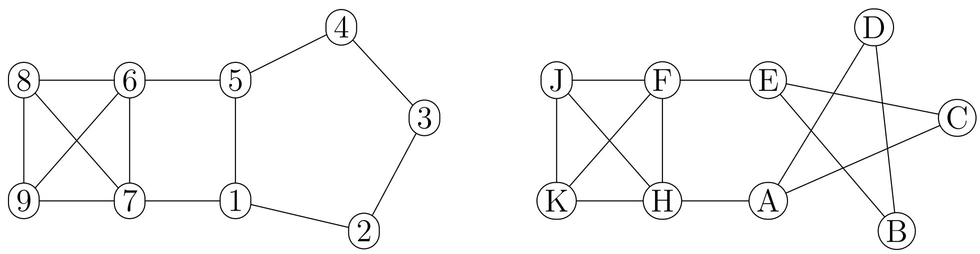
Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X Graph Y



Solution: No, they are not isomorphic. Both graphs have three degree-2 nodes. In Graph X one degree-2 node (3) has neighbors that are both degree 2. In Graph Y, one of the degree-2 nodes (C) has only degree-3 neighbors.

Alternatively, consider the two degree-3 nodes that are adjacent to a degree-2 node. In Graph X, these nodes (1 and 5) are adjacent. In Graph Y, these nodes (A and E) are not neighbors.

Alternatively, each graph contains a K_4 and a C_5 that don't overlap. Look at where the two connect. In Graph X, they connect at adjacent nodes. In graph Y, they connect at nodes that aren't neighbors.

2. (5 points) Use the pigeonhole principle to briefly explain why a graph with n nodes ($n \geq 2$) must have two nodes with the same degree. Hint: if one node has degree 0, what is the maximum degree of any other node? How many possible degree values are there?

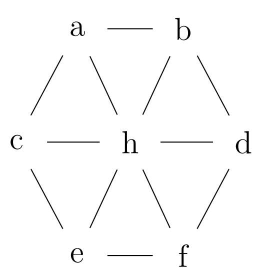
Solution: Node degrees range from 0 to $n-1$. However, it's impossible to have both a node with degree 0 and a node with degree $n-1$. So the degrees in any specific graph must either be between 0 and $n-2$, or else between 1 and $n-1$. In both cases, there are only $n-1$ distinct degrees. But there are n nodes. So two nodes must have the same degree.

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: 12. There are 6 choices for how to map node a. Then node b can map to either of the two adjacent nodes. After that, the rest of the mapping is forced.

2. (5 points) Complete this statement of the Handshaking Theorem.

For any graph G with set of nodes V and set of edges E, ...

Solution: The sum of the degrees of all the nodes is equal to twice the number of edges.

CS 173, Spring 18 Examlet 6, Part B

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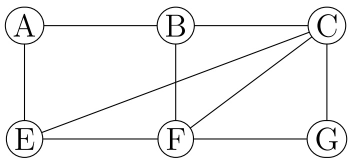
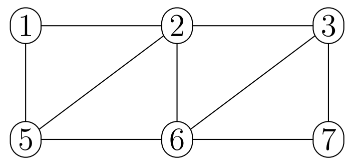
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



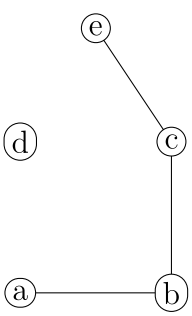
Graph Y

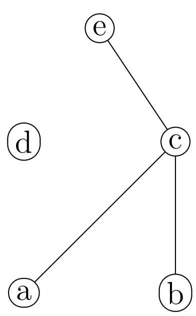


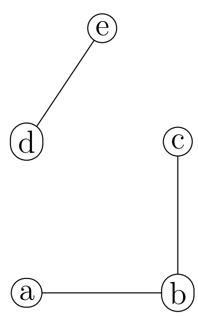
Solution: No, these are not isomorphic. In graph Y, both of the degree-2 nodes are in 3-cycles. In graph X, one of them (A) is not in a 3-cycle.

2. (5 points) Show three distinct (i.e. not isomorphic) graphs, each of which has five nodes, three edges, and no cycles.

Solution:







CS 173, Spring 18 Examlet 6, Part B

Name:____

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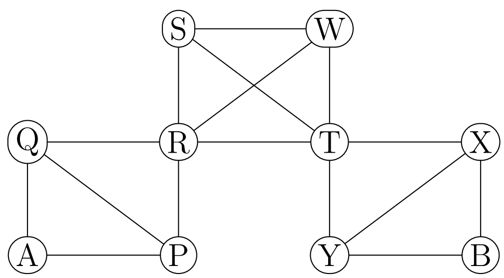
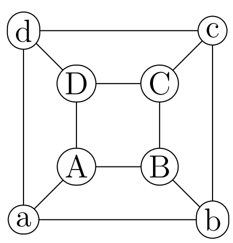
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: There are two choices for mapping R: to itself or to T. After that, Q and P can be interchanged (or not). And, X and Y can be interchanged (or not). Less obviously, S and W can also be interchanged. So there are $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$ isomorphisms.

2. (5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. Put nodes a, c, B, and D into one set and nodes A, C, b, and d into the other set.

Name:____

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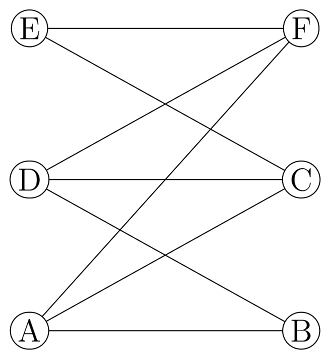
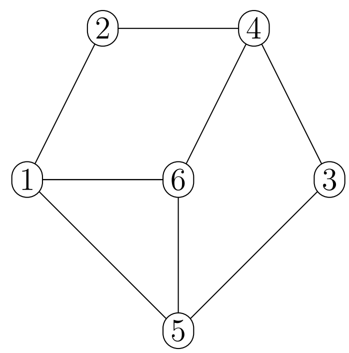
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

   Graph X

   Graph Y

   Solution: No, they aren't isomorphic. Graph Y has a 5-cycle but graph X has only even cycles. Also, one of the degree-3 nodes in graph Y is connected to three degree-3 nodes, but in graph X each degree-3 node is connected to only two other degree-3 nodes.

2. (5 points) Does the complete graph K_8 have an Euler circuit? Briefly justify your answer.

   Solution: No. Each node has degree 7, which is odd. You can't find an Euler circuit if there are any nodes with odd degree.

CS 173, Spring 18 Examlet 6, Part B

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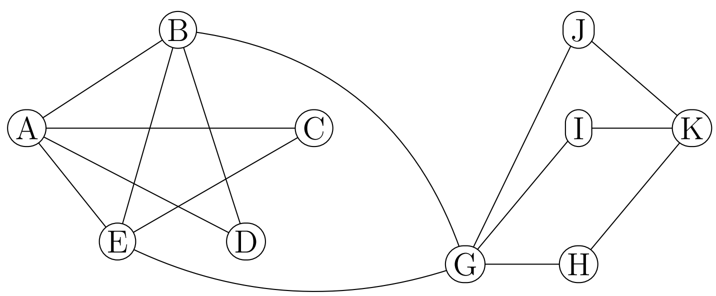
1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes A, G, and K must map onto themselves.

B and E can swap (2 choices). This fixes C and D. I, J, and H can be permuted (6 choices). So there are a total of $2 \cdot 6 = 12$ choices.

2. (5 points) If G is a graph, its complement G' has the same nodes as G but G' has an edge between nodes x and y if and only G does not have an edge between x and y. Give a succinct high-level description of the complement of W_5 (5-cycle joined to a hub node). Briefly justify or show work.

Solution: Suppose we label the hub node h and the rim nodes as a, b, c, d, and e. In the complement, we have edges ac, ad, bd, be, and ce. Rearranging these edges gives us: ac, ce, eb, bd, da. So we have a five-cycle, plus an isolated node (h).