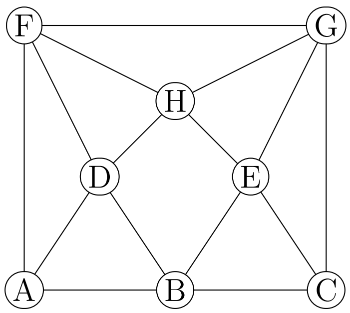
CS 173, Spring 18 Examlet 7, Part B

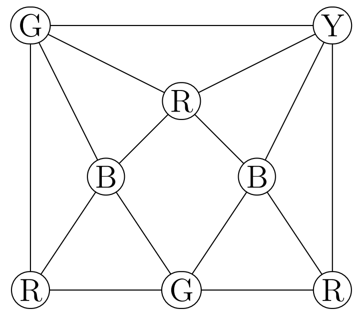
Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: This graph has chromatic number four. The picture above shows that four colors are enough. If you delete node H (and all its edges), you get the special graph presented towards the end of section 10.3 in the textbook, which we know to require four colors.

Alternatively, you can directly argue the lower bound as follows. Suppose we try to color this with three colors. Color the triangle A, B, D with R, G, B respectively. Then F must have color G. Nodes E and C must have colors R and B in some order. But then node G has neighbors with all three colors, so we're stuck. Therefore three colors is not enough.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with no cycles and at least one edge	1 ☐	2 ☑ √	3 ☐	can't tell ☐
15 guests are invited to brunch. Each guest will eat at least two buns. 20 is _____ on how many buns we will need.	an upper bound on ☐	a lower bound on ☑ √	exactly ☐	not a bound on ☐
$\sum_{k=0}^{n-1} \frac{1}{2^k}$	$1-(\frac{1}{2})^{n-1}$ ☐	$2-(\frac{1}{2})^{n}$ ☐	$1-(\frac{1}{2})^{n}$ ☐	$2-(\frac{1}{2})^{n-1}$ ☑ √

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} \ : \ |x+1| \leq 2\}$$
$$B = \{w \in \mathbb{R} \ : \ w^2 + 2w - 3 \leq 0\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution: $A \subseteq B$: Let x be a real number and suppose $x \in A$. Then $|x+1| \leq 2$. Therefore, $-2 \leq x+1 \leq 2$ so $-3 \leq x \leq 1$. Therefore $x+3 \geq 0$ and $x-1 \leq 0$. So $x^2+2x-3 = (x+3)(x-1) \leq 0$. So $x \in B$.

$B \subseteq A$: Let x be a real number and suppose $x \in B$. Then $x^2 + 2x - 3 \leq 0$. Factoring this polynomial, we get $(x+3)(x-1) \leq 0$. So $(x+3)$ and $(x-1)$ must have opposite signs. Since $x+3 > x-1$, it must be the case that $x+3 \geq 0$ and $x-1 \leq 0$. Therefore, $-3 \leq x \leq 1$. So $-2 \leq x+1 \leq 2$. So $|x+1| \leq 2$, and therefore $x \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p} i$	$\frac{p(p-1)}{2}$ ☐	$\frac{(p-1)^2}{2}$ ☐	$\frac{p(p+1)}{2}$ ☑ √	$\frac{(p-1)(p+1)}{2}$ ☐
Chromatic number of C_n.	2 ☐	3 ☐	≤ 3 ☑ √	≤ 4 ☐

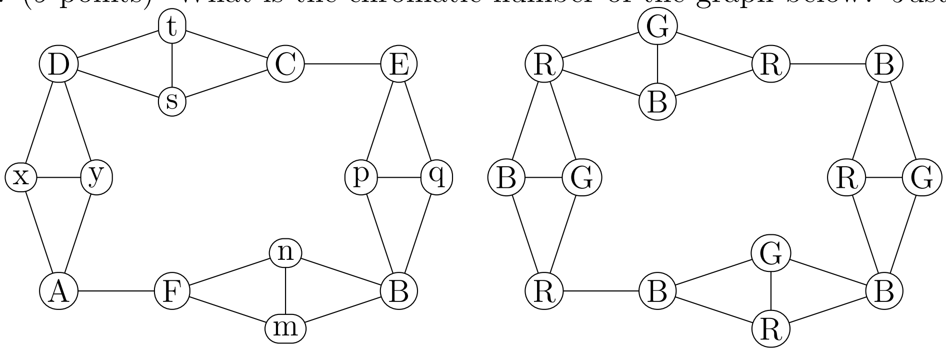
CS 173, Spring 18 Examlet 7, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number is three. The picture above shows that it can be colored with three colors (upper bound). Since it contains triangles, we also have a lower bound of three.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n-1} \frac{1}{2^k}$ $1-(\frac{1}{2})^n$ ☐ $2-(\frac{1}{2})^n$ ☐ $1-(\frac{1}{2})^{n-1}$ ☑ $2-(\frac{1}{2})^{n-1}$ ☐

10 guests are invited to brunch. Each guest will eat at least two buns. 30 is _____ on how many buns we will need.

an upper bound on ☐ exactly ☐
a lower bound on ☐ not a bound on ☑

Chromatic number of a graph with maximum vertex degree D

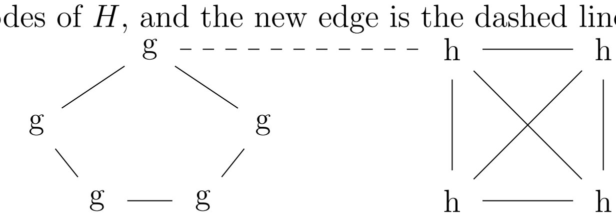
$= D$ ☐ $= D+1$ ☐
$\leq D+1$ ☑ $\geq D+1$ ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (11 points) If G is a graph, recall that $\chi(G)$ is its chromatic number. Suppose that G is a graph and H is another graph, not connected to G. Now, create a new graph T which consists of a copy of G, a copy of H, and a new edge that connects some node of G to some node of H. For example, suppose that G is C_5 and H is K_4. Then T might look as follows, where g marks nodes of G and h marks nodes of H, and the new edge is the dashed line.



Describe how $\chi(T)$ is related to $\chi(G)$ and $\chi(H)$, justifying your answer. Your answer should handle any choice for G and H.

Solution: $\chi(T) = \max(\chi(G), \chi(H), 2)$

Lower bound: G is a subgraph of T, so $\chi(T) \geq \chi(G)$. Similarly, $\chi(T) \geq \chi(H)$. We also know that $\chi(T) \geq 2$, because T contains at least one edge: the one connecting G to H. So $\chi(T) \geq \max(\chi(G), \chi(H), 2)$.

Upper bound: Suppose that $k = \max(\chi(G), \chi(H), 2)$. We can color G with k colors, because $k \geq \chi(G)$. Similarly, we can color H with the same k colors.

Now, suppose that x and y are the nodes connected by the new edge. If we have already assigned them different colors, then we are done.

If x and y have the same color, we must swap two of the color labels in H's coloring. That is, if x and y have color C, we find some other color D in our set. This second color D always exists because we've forced k to be at least 2. In the graph H, we then relabel each C node with D and each D node with C. Now x and y have different colors.

Now we have a coloring of T with k colors. So So $\chi(T) \leq k = \max(\chi(G), \chi(H), 2)$.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=0}^{n-1} 2^k$ — $2^n - 2$ ☐ $2^n - 1$ ☑ $2^{n-1} - 1$ ☐ $2^{n+1} - 1$ ☐

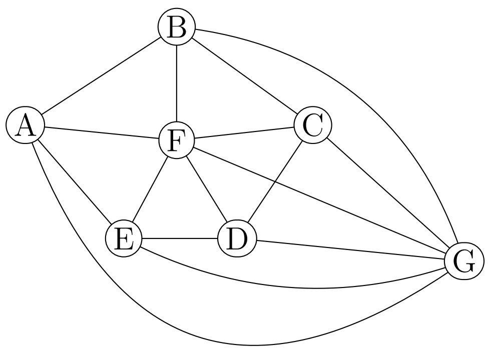
All elements of X are also elements of M. — $M = X$ ☐ $M \subseteq X$ ☐ $X \subseteq M$ ☑

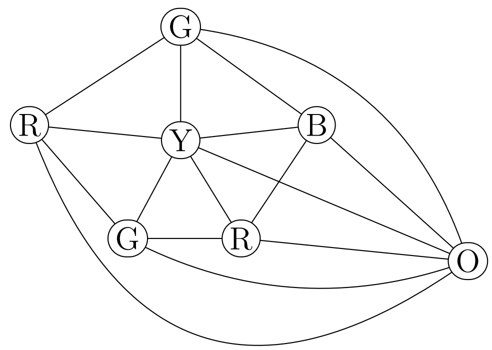
Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors. Then the node G is connected to all six nodes in the W_5, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} \frac{1}{2^k}$ $1-(\frac{1}{2})^{n-1}$ ☐ $2-(\frac{1}{2})^{n}$ ☐ $1-(\frac{1}{2})^{n}$ ☑ $2-(\frac{1}{2})^{n-1}$ ☐

Graph H has 6 nodes. 7 is ____ the chromatic number of H.
- an upper bound on ☑
- exactly ☐
- a lower bound on ☐
- not a bound on ☐

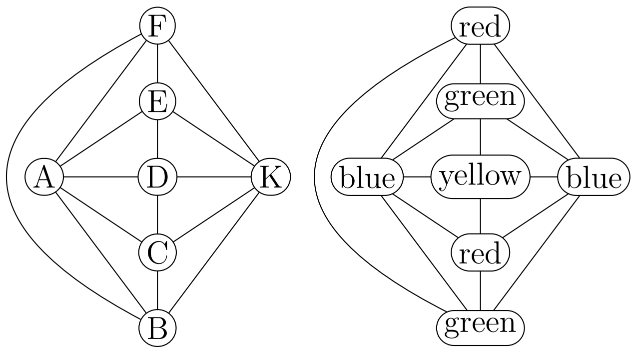
Chromatic number of G $\mathcal{C}(G)$ ☐ $\phi(G)$ ☐ $\chi(G)$ ☑ $\| G \|$ ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number is four. The picture shows the graph colored with four colors (upper bound). For the lower bound, notice that it contains a W_5: the rim is BCDEF and the hub is A.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} k$

$\sum_{p=1}^{n}(n-p+1)$	☑	$\sum_{p=1}^{n}(n-p)$	☐	$\sum_{p=0}^{n}(n-p)$	☐	$\sum_{p=1}^{n+1}(n-p)$	☐

10 students drove home in John's van. 10 is _____ how many students the van can carry.

an upper bound on	☐	exactly	☐
a lower bound on	☑	not a bound on	☐

Chromatic number of a graph (with at least one node) and no edges.

1	☑	2	☐	3	☐	can't tell	☐