CS 173, Spring 18 Examlet 8, Part A

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(20 points) Let x be a non-zero real number such that $x + \frac{1}{x}$ is an integer. Use (strong) induction to prove that $x^n + \frac{1}{x^n}$ is an integer, for any natural number n.

Hint: $(a^n + b^n)(a + b) = (a^{n+1} + b^{n+1}) + ab(a^{n-1} + b^{n-1})$, for any real numbers a and b.

Let x be a non-zero real number such that $x + \frac{1}{x}$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $h : \mathbb{Z}^+ \to \mathbb{Z}^+$ is defined by

$h(1) = 1$ $\quad h(2) = 7$

$h(n+1) = 7h(n) - 12h(n-1)$ for all $n \geq 2$

Use (strong) induction to prove that $h(n) = 4^n - 3^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that F_n is the nth Fibonacci number, and the positive Fibonacci numbers start with $F_1 = F_2 = 1$. Use (strong) induction to prove the following claim:

> Claim: Every positive integer can be written as the sum of (one or more) distinct Fibonacci numbers.

Hints: You can assume that the Fibonacci numbers are strictly increasing starting with F_1. To write x as the sum of Fibonacci numbers, start by including the largest Fibonacci number F_p such that $F_p \leq x$. (And therefore $x < F_{p+1}$.) How large is the remaining part of x?

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove the following claim:

> For any positive integer $n \geq 2$, if G is a graph with n nodes and more than $(n-1)(n-2)/2$ edges, then G is connected.

Hint: pick a node x. Perhaps x is connected to all the other nodes. If not, remove x to create a smaller graph H. What is the smallest number of edges that could remain in H? Notice that H has too few nodes to contain all the edges in G, so there is an edge from x to H.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Recall that F_n is the nth Fibonacci number, and these start with $F_0 = 0$, $F_1 = 1$. Use (strong) induction to prove the following claim:

Claim: $F_{n-1}F_{n+1} - (F_n)^2 = (-1)^n$ for any positive integer n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Let function $f : \mathbb{N} \to \mathbb{Z}$ be defined by

$f(0) = 3$

$f(1) = 9$

$f(n) = f(n-1) + 2f(n-2)$, for $n \geq 2$

Use (strong) induction to prove that $f(n) = 4 \cdot 2^n + (-1)^{n-1}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: