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(10 points) Suppose we have a function F defined (for n a power of 2) by

$$\begin{aligned} F(2) &= 17 \\ F(n) &= 3F(n/2), \text{ for } n \geq 4 \end{aligned}$$

Use unrolling to find the closed form for F. Show your work and simplify your answer.

Solution:

$$\begin{aligned} F(n) &= 3F(n/2) = 3(3F(n/4)) = 3(3(3(F(n/2^3)))) \\ &= 3^3F(n/2^3) \\ &= 3^kF(n/2^k) \end{aligned}$$

We'll hit the base case when $n/2^k = 2$, i.e. $n = 2^{k+1}$, i.e. $k+1 = \log_2 n$, $k = \log_2 n - 1$. Substituting this value into the above equation, we get

$$\begin{aligned} F(n) &= 3^{\log_2 n - 1} \cdot 17 \\ &= 17/3 \cdot 3^{\log_2 n} = 17/3 \cdot 3^{\log_3 n \log_2 3} \\ &= 17/3 n^{\log_2 3} \end{aligned}$$

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + d \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

Solution:

$$\begin{aligned} g(n) &= 4g(n/2) + d \\ &= 4(4g(n/2^2 + d) + d \\ &= 4(4(4g(n/2^3) + d) + d) + d \\ &= 4^3 g(n/2^3) + 21d \end{aligned}$$

2. (2 points) Check the (single) box that best characterizes each item.

$f(n) = n!$ can be defined recursively by $f(0) = 1$, and $f(n+1) = (n+1)f(n)$ for all integers ...	$n \geq 0$ ☑ (√)	$n \geq 1$ ☐	$n \geq 2$ ☐

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1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(9) &= 5 \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 27 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 3^k g(n/3^k) + kn$$

Finish finding the closed form for g. Show your work and simplify your answer.

Solution:

To find the value of k at the base case, set $n/3^k = 9$. Then $n = 3^{k+2}$, so $k + 2 = \log_3 n$, so $k + 2 = \log_3 n - 2$ Substituting this into the above, we get:

$$\begin{aligned} g(n) &= 3^{log_3 n - 2} g(9) + (log_3 n - 2)n \\ &= 3^{log_3 n} 3^{-2} 5 + n \log_3 n - 2n) \\ &= \frac{5}{9}n + n\log_3 n - 2n = n\log_3 n - \frac{13}{9}n \end{aligned}$$

2. (2 points) Suppose that G_0 is the graph consisting of a single vertex. Also suppose that the graph G_n consists of a copy of G_{n-1} plus an extra vertex v and edges joining v to each vertex in G_{n-1}. Give a clear picture or precise description of G_4.

Solution: This is a recursive construction of all the complete graphs, except for the indexing being off by one. So G_4 is just K_5.

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(10 points) Suppose we have a function F defined (for n a power of 3) by

$$\begin{aligned} F(1) &= 5 \\ F(n) &= 3F(n/3) + 7 \text{ for } n \geq 3 \end{aligned}$$

Your partner has already figured out that

$$F(n) = 3^k F(n/3^k) + 7\sum_{p=0}^{k-1} 3^p$$

Finish finding the closed form for F. Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$

Solution:

To find the value of k at the base case, set $n/3^k = 1$. Then $n = 3^k$, so $k = \log_3 n$. Substituting this into the above, we get

$$\begin{aligned} F(n) &= 3^{\log_3 n} \cdot 5 + 7 \sum_{p=0}^{\log_3 n - 1} 3^p \\ &= 5n + 7\frac{3^{log_3 n} - 1}{3-1} \\ &= 5n + 7\frac{n-1}{3-1} = 5n + \frac{7(n-1)}{2} \end{aligned}$$

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

Solution:

$$\begin{aligned} g(n) &= 4g(n/2) + n \\ &= 4(4g(n/4) + n/2) + n \\ &= 4(4(4g(n/8) + n/4) + n/2) + n \\ &= 4^3 g(n/8) + 4n + 2n + n \\ &= 4^3 g(n/8) + 7n \end{aligned}$$

2. (2 points) Check the (single) box that best characterizes each item.

The diameter of the 4-dimensional hypercube Q_4	1 ☐	2 ☐	4 ☑	16 ☐

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1. (8 points) Suppose we have a function f defined (for n a power of 2) by

$$
\begin{aligned}
f(1) &= 5 \\
f(n) &= 3f(n/2) + n^2 \text{ for } n \geq 2
\end{aligned}
$$

Express $f(n)$ in terms of $f(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

Solution:

$$
\begin{aligned}
f(n) &= 3f(n/2) + n^2 \\
&= 3(3f(n/4) + (n/2)^2) + n^2 \\
&= 3(3(3f(n/8) + (n/4)^2) + (n/2)^2) + n^2 \\
&= 27f(n/2^3) + (3/4)n^2 + (9/16)n^2 + n^2 \\
&= 27f(n/2^3) + (2 + 5/16)n^2
\end{aligned}
$$

2. (2 points) Check the (single) box that best characterizes each item.

The n-dimensional hypercube Q_n has an Euler circuit.	always ☐	sometimes ☑	never ☐