

Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(10 points) Suppose we have a function F defined (for n a power of 2) by

$$F(2) = 17$$

$$F(n) = 3F(n/2), \text{ for } n \geq 4$$

Use unrolling to find the closed form for F . Show your work and simplify your answer.

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + d \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

$f(n) = n!$ can be defined recursively by
 $f(0) = 1$, and $f(n+1) = (n+1)f(n)$
 for all integers ...

$n \geq 0$ ☐

$n \geq 1$ ☐

$n \geq 2$ ☐

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1. (8 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(9) &= 5 \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 27 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 3^k g(n/3^k) + kn$$

Finish finding the closed form for g . Show your work and simplify your answer.

2. (2 points) Suppose that G_0 is the graph consisting of a single vertex. Also suppose that the graph G_n consists of a copy of G_{n-1} plus an extra vertex v and edges joining v to each vertex in G_{n-1} . Give a clear picture or precise description of G_4 .

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(10 points) Suppose we have a function F defined (for n a power of 3) by

$$\begin{aligned} F(1) &= 5 \\ F(n) &= 3F(n/3) + 7 \text{ for } n \geq 3 \end{aligned}$$

Your partner has already figured out that

$$F(n) = 3^k F(n/3^k) + 7 \sum_{p=0}^{k-1} 3^p$$

Finish finding the closed form for F . Show your work and simplify your answer. Recall the following useful closed form (for $r \neq 1$): $\sum_{k=0}^n r^k = \frac{r^{n+1} - 1}{r - 1}$

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1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The diameter of the
4-dimensional hypercube Q_4

1

☐

2

☐

4

☐

16

☐

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1. (8 points) Suppose we have a function f defined (for n a power of 2) by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The n -dimensional

hypercube Q_n has an Euler circuit.

always ☐ sometimes ☐ never ☐