Name:

NetID: **Lecture:**   A   B

Discussion: **Thursday**   **Friday**   **9**   **10**   **11**   **12**   **1**   **2**   **3**   **4**   **5**   **6**

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `ababab` that match this grammar.

$$S \;\to\; a\,N \;\mid\; a\,N\,S$$
$$N \;\to\; b\,a\,N \;\mid\; b$$

Solution:
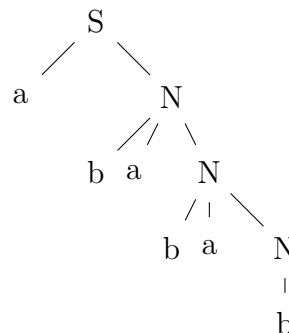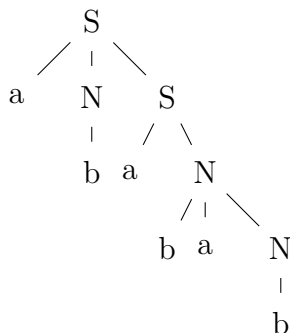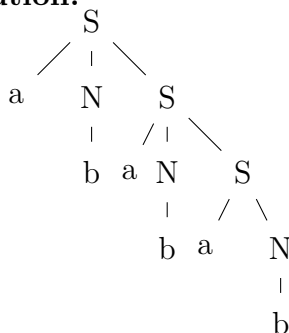
2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a descendent
of itself.   always $\boxed{\checkmark}$ sometimes $\square$ never $\square$

The number of nodes in a
full complete binary tree of height h

$\geq 2^h$ $\square$ $2^{h+1} - 1$ $\boxed{\checkmark}$

$\leq 2^{h+1} - 1$ $\square$ $\geq 2^{h+1} - 1$ $\square$

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1. (8 points)   Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \;\rightarrow\; S\,S\,a \;\mid\; c\,S \;\mid\; c\,c$$

```
      S                    S                          S                      S
     / \                  / \                        / \                    / \
    c   S                c   S                      c   S                  c   S
                            / \                        /|\                    / \
                           c   c                      S S  a                 c   S
                                                     /|  |\
                                                    c c  c c
```
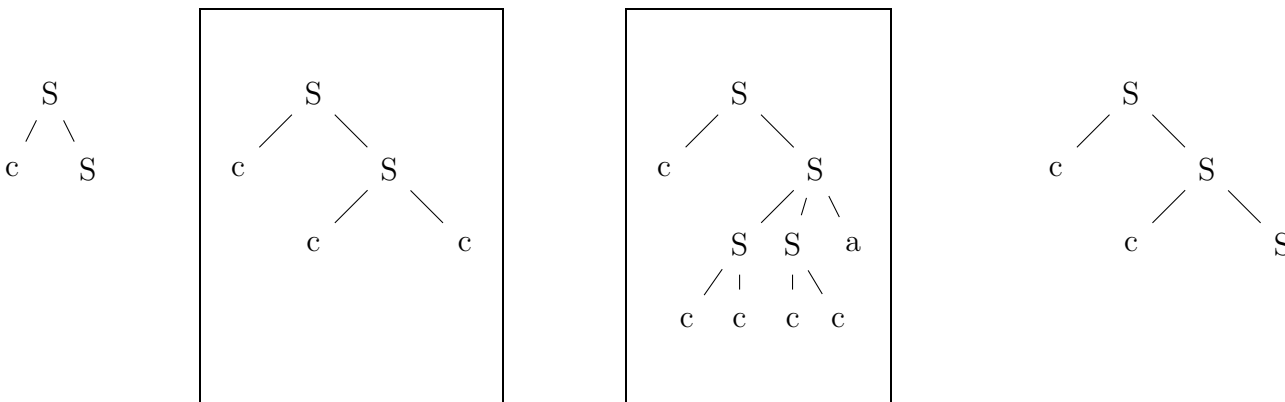
2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at most $2^{h+1} - 1$ nodes.   true ☑   false ☐

A tree with n edges has ____ nodes.

$n-1$ ☐ n ☐

$n+1$ ☑ $n/2$ ☐

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1. (8 points) Consider the following grammar G

$$S \rightarrow a\ S\ b \mid b\ S\ b \mid a \mid b$$

S is the only start symbol. The terminal symbols are a and b.

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

aabaaba
Solution:
This is impossible. Strings produced by G have to end in a b, unless they are length 1.

aababaa
Solution:
This is impossible. Strings produced by G have to end in a b, unless they are length 1.

2. (4 points) Check the (single) box that best characterizes each item.

The root node of a tree is an internal node   always ☐   sometimes ☑   never ☐

The level of the root node in a tree of height h. -1 ☐ 0 ☑ 1 ☐ $h-1$ ☐ h ☐

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1. (8 points) Give a context-free grammar that generates all strings of the form a^+b^+. That is, all strings that consist of a sequence of one or more a's followed by a sequence of one or more b's.

   Solution:

   $S \rightarrow A\ B$

   $A \rightarrow a\ A\ \mid\ a$

   $B \rightarrow b\ B\ \mid\ b$

2. (4 points) Check the (single) box that best characterizes each item.

   Number of non-empty bit strings of length k.
      2^k ☑ $2^k - 1$ ☐ 2^{k-1} ☐ k ☐

   The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same.
      n ☐ $2n$ ☐ $\frac{n(n-1)}{2}$ ☑

      $n(n-1)$ ☐ n^2 ☐ $\frac{n(n+1)}{2}$ ☐
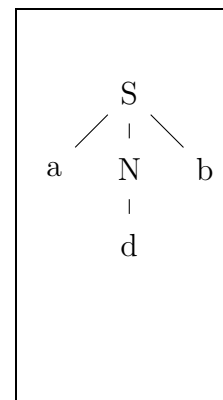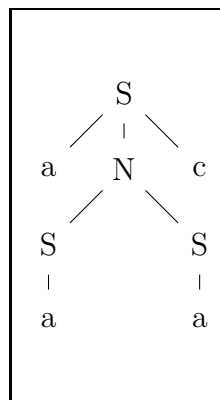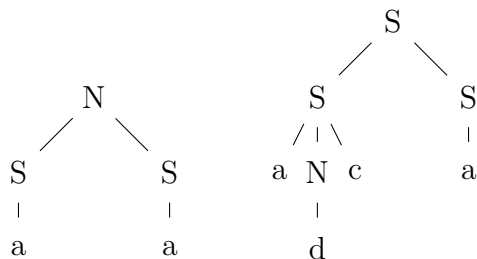
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1. (8 points)   Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$
\begin{aligned}
S &\rightarrow & a\ N\ b\ |\ a\ N\ c\ |\ a \\
N &\rightarrow & S\ S\ |\ d
\end{aligned}
$$

```
          S                    S                      S                     S
        /   \                /   \                  / | \                 / | \
      N       S            S       S              a   N   c             a   N   b
    /   \     |          / | \     |                /   \                   |
   S     S    a        a  N  c     a               S     S                  d
   |     |               |                         |     |
   a     a               d                         a     a
```

2. (4 points) Check the (single) box that best characterizes each item.

A full m-ary tree with i internal nodes has _____ nodes total.

$mi - 1$ ☐ mi ☐

$mi + 1$ ☑ $\leq mi + 1$ ☐

Height of a binary tree with 2^n nodes. $\leq n - 1$ ☐ $\leq n$ ☐ $\leq 2^n$ ☐ $\leq 2^n - 1$ ☑
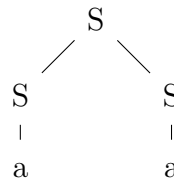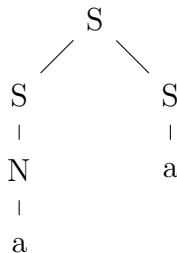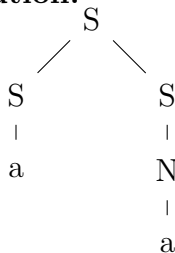
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1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string `aa` that match this grammar.

$$S \rightarrow S\,S \mid N \mid a$$
$$N \rightarrow a$$

Solution:

```
     S                    S                    S
    / \                  / \                  / \
   S   S                S   S                S   S
   |   |                |   |                |   |
   a   N                N   a                a   a
       |                |
       a                a
```

2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a proper ancestor
of itself.   always ☐   sometimes ☐   never ☑

Removing an edge from a tree
(with at least one edge) produces
two trees.   always ☑   sometimes ☐   never ☐