

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a$  and  $b$ . Draw three parse trees for the string **ababab** that match this grammar.

$$\begin{aligned} S &\rightarrow a N \mid a N S \\ N &\rightarrow b a N \mid b \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a descendent  
of itself.

always

☐

sometimes

☐

never

☐

The number of nodes in a  
full complete binary tree of height  $h$

$\geq 2^h$

☐

$2^{h+1} - 1$

☐

$\leq 2^{h+1} - 1$

☐

$\geq 2^{h+1} - 1$

☐

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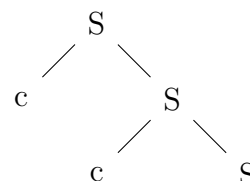
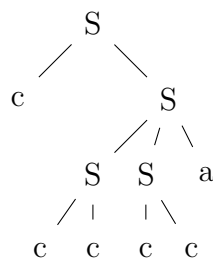
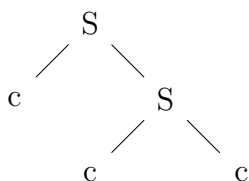
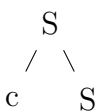
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1. (8 points) Here is a grammar, with start variable  $S$  and terminals  $a$  and  $c$ . Circle the trees that match the grammar.

$$S \rightarrow S S a \mid c S \mid c c$$



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height  $h$  has at most  $2^{h+1} - 1$  nodes.

true

☐

false

☐

A tree with  $n$  edges has \_\_\_\_\_ nodes.

 $n - 1$ 
☐
 $n$ 
☐
 $n + 1$ 
☐
 $n/2$ 
☐

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1. (8 points) Consider the following grammar  $G$

$$S \rightarrow a S b \mid b S b \mid a \mid b$$

$S$  is the only start symbol. The terminal symbols are  $a$  and  $b$ .

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar  $G$  whose leaves have this sequence of labels, or else explain briefly why  $G$  cannot generate this sequence of leaf labels.

aabaaba

aababaa

2. (4 points) Check the (single) box that best characterizes each item.

The root node of a tree is an internal node

always

☐

sometimes

☐

never

☐

The level of the root node in a tree of height  $h$ .

-1

☐

0

☐

1

☐ $h - 1$ ☐ $h$ ☐

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1. (8 points) Give a context-free grammar that generates all strings of the form  $a^+b^+$ . That is, all strings that consist of a sequence of one or more a's followed by a sequence of one or more b's.

2. (4 points) Check the (single) box that best characterizes each item.

Number of non-empty bit strings of length  $k$ .

 $2^k$  ☐ $2^k - 1$  ☐ $2^{k-1}$  ☐ $k$  ☐

The number of paths between two distinct nodes in an  $n$ -node tree. Paths in opposite directions count as the same.

 $n$  ☐ $2n$  ☐ $\frac{n(n-1)}{2}$  ☐ $n(n-1)$  ☐ $n^2$  ☐ $\frac{n(n+1)}{2}$  ☐

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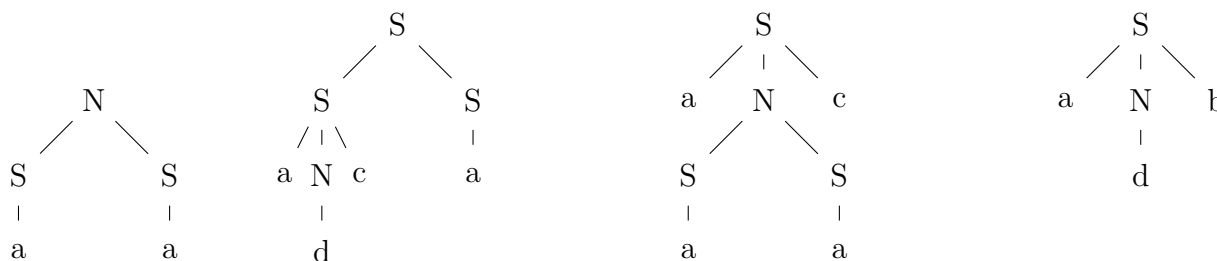
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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbols  $a$ ,  $b$ ,  $c$ , and  $d$ . Circle the trees that match the grammar.

$$S \rightarrow a N b \mid a N c \mid a$$

$$N \rightarrow S S \mid d$$



2. (4 points) Check the (single) box that best characterizes each item.

A full  $m$ -ary tree with  $i$   
internal nodes has \_\_\_\_\_  
nodes total.

$mi - 1$  ☐

$mi$  ☐

$mi + 1$  ☐

$\leq mi + 1$  ☐

Height of a binary  
tree with  $2^n$  nodes.

$\leq n - 1$  ☐

$\leq n$  ☐

$\leq 2^n$  ☐

$\leq 2^n - 1$  ☐

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1. (8 points) Here is a grammar with start symbol  $S$  and terminal symbol  $a$ . Draw three parse trees for the string  $aa$  that match this grammar.

$$\begin{aligned} S &\rightarrow S S \mid N \mid a \\ N &\rightarrow a \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

A tree node is a proper ancestor of itself.

always

☐

sometimes

☐

never

☐

Removing an edge from a tree (with at least one edge) produces two trees.

always

☐

sometimes

☐

never

☐