CS 173, Spring 18 Examlet 9, Part A

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(18 points) Recall that F_n is the nth Fibonacci number, and these start with $F_0 = 0$, $F_1 = 1$.

Let T_n be the number of bit strings of length n that don't contain any consecutive zeros. E.g. when counting strings of length 6, we include 010110, but not 101001. Prove that $T_n = F_{n+2}$ for any natural number n. Hint: if w is a string with no consecutive zeros, either $w = 1x$, where x is a shorter string, or $w = 01y$, where y is a shorter string.

Solution: The induction variable is named **n** and it is the **length** of/in the string.

Base Case(s): At $n = 0$, we have only the empty string ϵ. So $T_0 = 1 = F_2$.

At $n = 1$, we have two strings 0 and 1. So $T_1 = 2 = F_3$.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]: Prove that $T_n = F_{n+2}$ for $n = 0, 1, \ldots, k-1$.

Inductive Step: Now consider $n = k$. We need to calculate T_k which is the number of bit strings of length k with no consecutive zeros. Following the hint, these strings come in two mutually-exclusive types:

Type 1: strings of the form $1x$, where x is a string of length $k-1$. By the induction hypothesis, there are F_{k+1} choices for x. So there are F_{k+1} choices for $1x$.

Type 2: strings of the form $01y$, where y is a string of length $k-2$. By the induction hypothesis, there are F_k choices for y. So there are F_k choices for $1y$.

T_k is equal to the number of strings of type 1 plus the number of strings of type 2. So $T_k = F_{k+1} + F_k$. By the definition of Fibonacci numbers, this is equal to F_{k+2}. So $T_k = F_{k+2}$, which is what we needed to show.

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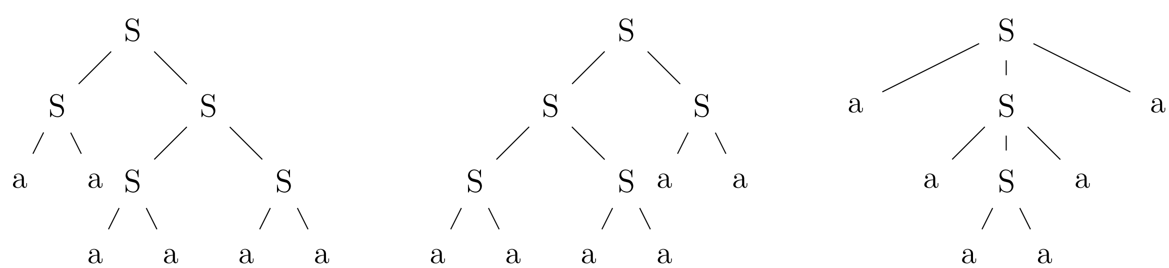
1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string `aaaaaa` that match this grammar.

$$S \rightarrow SS \mid aSa \mid aa$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a 3-ary tree of height h	3^h ☐	$\leq 3^h$ ☑	$\frac{1}{2}(3^{h+1}-1)$ ☐	$3^{h+1}-1$ ☐	
The level of a leaf node in a tree of height h.	0 ☐	1 ☐	$h-1$ ☐	$\leq h$ ☑	h ☐