CS 173, Spring 18 Examlet 10, Part A

Name:____

NetID:____________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Use (strong) induction to prove the following claim:

Claim: For any natural number n and any real number $x > -1$, $(1+x)^n \geq 1 + nx$.

Let x be a real number with $x > -1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq \sqrt{n}$

You may use the fact that $\sqrt{n+1} \geq \sqrt{n}$ for any natural number n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$f(1) = f(2) = 1$

$f(n) = \frac{1}{2}f(n-1) + \frac{1}{f(n-2)}$

Use (strong) induction to prove that $1 \leq f(n) \leq 2$ for all positive integers n.

Hint: prove both inequalities together using one induction.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Recall the following fact about real numbers

Triangle Inequality: For any real numbers x and y, $|x+y| \leq |x| + |y|$.

Use this fact and (strong) induction to prove the following claim:

Claim: For any real numbers $x_1, x_2, \ldots, x_n$ ($n \geq 2$), $|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: $\prod_{p=1}^{n} \frac{2p-1}{2p} < \frac{1}{\sqrt{2n+1}}$ for all integers $n \geq 1$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hints: Work backwards from the goal, then rewrite into logical order. Try squaring both sides. For positive numbers a and b, $a < b$ if and only if $\sqrt{a} < \sqrt{b}$.

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(15 points) Suppose that $0 < q < \frac{1}{2}$. Use (strong) induction to prove the following claim:

Claim: $(1+q)^n \leq 1 + 2^n q$, for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Notice that $1 \leq 2^{n-1}$ for any positive integer n.