Name:			

NetID:\_\_\_\_\_

Lecture: A B

Discussion: T

Thursday Friday

9 10

11

12

1

 $\mathbf{2}$ 

3

4 5

5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

$$T(1) = 7$$

$$T(n) = 2T\left(\frac{n}{4}\right) + n$$

- (a) The height:
- (b) Number of leaves:
- (c) Total work (sum of the nodes) at level k (please simplify):

Change of base formula:  $\log_b n = (\log_a n)(\log_b a)$ 

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if  $f(n) \ll g(n)$ .

42n!

 $7^n$ 

 $100 \log n$ 

 $n\log(n^7)$ 

 $2^{3n}$ 

 $\log(2^n)$ 

 $(n^3)^7$ 

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1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If  $g(x) \ll h(x)$ , then  $\log(g(x)) \ll \log(h(x))$ . Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$$T(1) = d$$
  
$$T(n) = T(n/2) + c$$

$$\Theta(\log n)$$
  
 $\Theta(n^2)$ 

$\Theta(\sqrt{n})$
$\Theta(n^3)$

$$\Theta(n)$$

$$\Theta(2^n)$$

$\Theta(n \log n)$
$\Theta(3^n)$

$$T(1) = d$$
  

$$T(n) = 2T(n-1) + c$$

$$\Theta(\log n)$$
  $\Theta(n^2)$ 





$\Theta(n \log n)$
$\Theta(3^n)$

 $2^n$  is

$\Theta(3^n)$		
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$$O(3^n)$$

neither of these

Suppose f and g produce only positive outputs and  $f(n) \ll g(n)$ . Will f(n) be  $\Theta(g(n))$ ?

no

perhaps

yes

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f is O(g) and g is O(h). Must f be O(h)? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$$T(1) = d$$
  

$$T(n) = T(n/2) + n$$

$$\Theta(\log n)$$

$$\Theta(n^2)$$

$\Theta(\sqrt{n})$
$\Omega(n^3)$

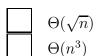
$$\begin{array}{c|c} & \Theta(n) \\ \hline & \Theta(2^n) \end{array}$$

$$\Theta(n \log n)$$
 
$$\Theta(3^n)$$

$$T(1) = d$$

$$T(n) = 3T(n-1) + c$$

$$\Theta(\log n)$$
  
 $\Theta(n^2)$ 



$$\Theta(n)$$

$$\Theta(2^n)$$

$\Theta(n \log n)$
$\Theta(3^n)$

 $n^{log_25}$  grows

faster than  $n^2$  at the same rate as  $n^2$ 

slower than  $n^2$ 

 $\mathbf{A}$ 

 $\mathbf{B}$ 

Suppose f(n) is O(g(n)). Will g(n) be O(f(n))?

no

perhaps

yes

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f(x) is O(h(x)) and g(x) is O(h(x)). Must f(x)g(x) be O(h(x)h(x))?

2. (8 points) Check the (single) box that best characterizes each item.

Suppose f(n) is O(g(n)). Will f(n) be  $\Theta(g(n))$ ?

no

perhaps

yes \_\_\_\_

 $\mathbf{B}$ 

 $17n^3$ 

 $\Theta(n^3)$ 

 $O(n^3)$ 

neither of these

T(1) = c $T(n) = 2T(n/2) + n^2$   $\Theta(\log n)$ 

 $\Theta(n^2)$ 

 $\Theta(\sqrt{n})$   $\Theta(n^3)$ 

 $\Theta(n \log n)$ 

 $\Theta(n \log n)$   $\Theta(3^n)$ 

T(1) = dT(n) = T(n/3) + c  $\Theta(\log n)$ 

 $\Theta(n^2)$ 

 $\Theta(n)$   $\Theta(2^n)$ 

 $\Theta(n \log n)$   $\Theta(3^n)$ 

 $\mathbf{B}$ 

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

$$T(1) = 1 T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

- (a) Value in each node at level k:
- (b) Total work (sum of the nodes) at level k (please simplify):
- (c) Sum of the work in all internal (non-leaf) nodes (please simplify):

Change of base formula:  $\log_b n = (\log_a n)(\log_b a)$ 

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if  $f(n) \ll g(n)$ .

 $n \log n$   $\log(n^{17})$   $\sqrt{n} + n! + 18$   $2^n$   $8n^2$   $8^{\log_8 n}$   $0.001n^3$ 

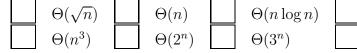
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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be O(g).

2. (8 points) Check the (single) box that best characterizes each item.

$$T(1) = c$$
  $\Theta(\log n)$   $\Theta(\sqrt{n})$   $\Theta(n)$   $\Theta(n)$ 



$$T(1) = d \qquad \qquad \Theta(\log n) \qquad \qquad \Theta(\sqrt{n}) \qquad \qquad \Theta(n \log n) \qquad \qquad \\ T(n) = 2T(n/2) + c \qquad \qquad \Theta(n^2) \qquad \qquad \Theta(n^3) \qquad \qquad \Theta(2^n) \qquad \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\ \Theta(3^n) \qquad \\$$

O(n!) neither of these

Suppose f and g produce only positive outputs and  $f(n) \ll g(n)$ . no perhaps yes Will f(n) be O(g(n))?