CS 173, Spring 18 Examlet 10, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(1) = 7$ $\qquad$ $T(n) = 2T\left(\frac{n}{4}\right) + n$

   (a) The height:

   (b) Number of leaves:

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\qquad$ 7^n $\qquad$ $100 \log n$ $\qquad$ $n \log(n^7)$ $\qquad$ 2^{3n} $\qquad$ $\log(2^n)$ $\qquad$ $(n^3)^7$

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (7 points) In class, Prof. Snape made the following claim about all functions g and h from the reals to the reals whose output values are always > 1. If $g(x) \ll h(x)$, then $\log(g(x)) \ll \log(h(x))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = 2T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
2^n is	$\Theta(3^n)$ ☐	$O(3^n)$ ☐	neither of these ☐	
Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $\Theta(g(n))$?	no ☐	perhaps ☐	yes ☐	

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f is $O(g)$ and g is $O(h)$. Must f be $O(h)$? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 3T(n-1) + c$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$n^{log_2 5}$ grows

faster than n^2 ☐ slower than n^2 ☐
at the same rate as n^2 ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$?

no ☐ perhaps ☐ yes ☐

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x)h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $O(g(n))$. Will $f(n)$ be $\Theta(g(n))$?	no	☐	perhaps	☐	yes	☐		
$17n^3$	$\Theta(n^3)$	☐	$O(n^3)$	☐	neither of these	☐		
$T(1) = c$ $T(n) = 2T(n/2) + n^2$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐
$T(1) = d$ $T(n) = T(n/3) + c$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Name:____

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(1) = 1$ $\qquad$ $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

   (a) Value in each node at level k:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) Sum of the work in all internal (non-leaf) nodes (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \log n$ $\qquad$ $\log(n^{17})$ $\qquad$ $\sqrt{n} + n! + 18$ $\qquad$ 2^n $\qquad$ $8n^2$ $\qquad$ $8^{\log_8 n}$ $\qquad$ $0.001n^3$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 3T(n/3) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

2^n $\Theta(n!)$ ☐ $O(n!)$ ☐ neither of these ☐

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$? no ☐ perhaps ☐ yes ☐