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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

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01 Hoist(a_1, \ldots, a_n): an array of n positive integers)
     if (n = 1) return 0
03
     else if (n = 2) return a_1 + a_2)
04
05
         p = |n/3|
06
         q = |2n/3|
         rv = max(Hoist(a_1, ..., a_p), Hoist(a_{q+1}, ..., a_n))
07
08
         for i=p to q
09
            rv = \max(rv, a_i + a_{i+1})
10
         return rv
```

1. (5 points) Let T(n) be the running time of Hoist. Give a recursive definition of T(n).

2. (3 points) What is the height of the recursion tree for T(n), assuming n is a power of 3?

3. (3 points) What is amount of work (aka sum of the values in the nodes) at level k of this tree?

4. (4 points) How many leaves does this recursion tree have? Simplify so that your answer is easy to compare to standard running times. Recall that  $\log_b x = \log_a x \log_b a$ .

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- 01 Weave $(a_1,..., a_n) \setminus$  input is an array of n integers
- $02 \qquad \text{for } i = 1 \text{ to } n 1$
- 03 min = i
- of for j = i to n
- if  $a_i < a_{min}$  then min = j
- swap $(a_i, a_{min})$  \\ interchange the values at positions i and min in the array
- 1. (3 points) If the input is 10, 5, 2, 3, 8, what are the array values after two iterations of the outer loop?

2. (3 points) Let T(n) be the number of times that line 5 is executed. Express T(n) using summation notation, directly following the structure of the code.

- 3. (3 points) Find an (exact) closed form for T(n). Show your work.
- 4. (3 points) What is the big-theta running time of Weave?
- 5. (3 points) Check the (single) box that best characterizes each item.

The running time of Karatsuba's algorithm is recursively defined by T(1) = d and T(n) =

$$2T(n/2) + cn$$
$$4T(n/2) + cn$$

3T(n/2) + cn	
4T(n/2) + c	

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01 Handl	$e(L_1, L_2: $ sorted list	s of integers	s)									
	if $(L_1 \text{ is empty})$	O	,									
03	return $L_2$											
04	else if $(L_2 \text{ is empty})$	r)										
05	return $L_1$											
06	else if $(\text{head}(L_1) <$	$= head(L_2)$	)									
07	return cons(h	$\operatorname{ead}(L_1)$ , H	andle	(rest(L	$(1), L_2)$	)						
08	else											
09	return cons(h	$\operatorname{ead}(L_2)$ , H	andle	$(L_1, res$	$\operatorname{t}(L_2)))$	)						
1. (5 points)	ad, rest, cons, and Suppose that $n$ is to Give a recursive d	the sum of t	he ler	ngths o						the 1	runni	ng time
2. (3 points)	What is the height	of the recu	ırsion	tree fo	or $T(n)$	)?						
3. (3 points)	What is amount of	f work (aka	sum	of the	values	in the	node	s) at l	evel	k of t	this t	ree?
4. (4 points)	What is the big-th	eta running	$g  ext{ time}$	e of Ha	ndle?							

01 Execute( $p_1$ , 02 if ( $n = 03$ ) 04 else 05 06 07	= 3) return the la $x = Execute($ $y = Execute($ $z = E$	argest of $d(p_1, p_2, p_3, p_4, \dots, (p_1, p_3, p_4, \dots, (p_1, p_2, p_4, \dots, x, y, z))$	ents, $n$ $(p_1, p_2), \dots, (p_n), (p_n), (p_n), (p_n), \dots, (p_n)$ me) t	$10$ $\geq 3)$ $d(p_1, p_1, p_2, p_3)$ $\downarrow \setminus 1$ $\downarrow \setminus 1$ the strain	removi removi removi ight-lin	$oxed{12}$ d $d(p_2,$ $\log p_1$ for $\log p_2$ for $\log p_3$	rom li rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
01 Execute( $p_1$ , 02 if ( $n = 0.03$ ) 04 else 05 06 07 08  The function $d(p, 0.05)$ Suppose 1. (5 points) Supp	, $p_n$ : list = 3) return the la x = Execute(y) y = Execute(y) z = Execute(y) return $\max(x)$ y = Execute(y)	of $n$ 2D point argest of $d(p_1, p_2, p_3, p_4, \dots, (p_1, p_3, p_4, \dots, (p_1, p_2, p_4, \dots, x, y, z)$ in constant ti	ents, $n$ $(p_1, p_2), \dots, (p_n), (p_n), (p_n), (p_n), \dots, (p_n)$ me) t	$\geq 3$ ) $d(p_1, p_1, p_2)$ $\downarrow \downarrow $	o <sub>3</sub> ), and removi removi removi	$d\ d(p_2,$ $\log\ p_1\ f$ $\log\ p_2\ f$ $\log\ p_3\ f$ $\log$ distance distance	$p_3$ )  rom lift rom lift ance l	ist tak ist tak ist tak	xes co xes co xes co	onstar onstar onstar vo poi	nt tim nt tim nt tim	ne ne ne and q
02 if $(n = 0.03)$ 04 else 05 c 06 c 07 c 08  The function $d(p, 0.05)$	= 3) return the la $x = Execute($ $y = Execute($ $z = E$	argest of $d(p_1, p_2, p_3, p_4, \dots, (p_1, p_3, p_4, \dots, (p_1, p_2, p_4, \dots, x, y, z))$	$(p_1, p_2),$ $(p_n),$ $(p_n),$ $(p_n),$ $(p_n),$ $(p_n),$	$d(p_1, p_1)$	removi removi removi ight-lin	$\log p_1$ for $p_2$ for $p_3$ for $p_$	rom li rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
03 04 else 05 5 06 5 07 5 08 The function $d(p, q)$	return the la $x = \text{Execute}(y) = \text{Execute}$	$(p_2, p_3, p_4, \dots (p_1, p_3, p_4, \dots (p_1, p_2, p_4, \dots (p_1, p_2, p_4, \dots (x, y, z))$	$(p_n)$ $(p_n)$ $(p_n)$ $(p_n)$	\\ 1 \\ 1 \\ 1	removi removi removi ight-lin	$\log p_1$ for $p_2$ for $p_3$ for $p_$	rom li rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
04 else $05$ $06$ $07$ $08$ The function $d(p, 0)$	x = Execute(y) = Execut	$(p_2, p_3, p_4, \dots (p_1, p_3, p_4, \dots (p_1, p_2, p_4, \dots (p_1, p_2, p_4, \dots (x, y, z))$	$(p_n)$ $(p_n)$ $(p_n)$ $(p_n)$	\\ 1 \\ 1 \\ 1	removi removi removi ight-lin	$\log p_1$ for $p_2$ for $p_3$ for $p_$	rom li rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
05 $06$ $07$ $08$ The function $d(p, q)$ 1. (5 points) Supp	y = Execute(z) $z = Execute(z)$ $z =$	$(p_1, p_3, p_4, \dots, (p_1, p_2, $	$(p_n)$ $(p_n)$ me) t	\\ ı\\ ı	removi removi .ight-lii	$\log p_2$ for $\log p_3$ for $\log p_3$ for $\log p_3$ for $\log p_3$	rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
06 $07$ $08$ The function $d(p, q)$ 1. (5 points) Supp	y = Execute(z) $z = Execute(z)$ $z =$	$(p_1, p_3, p_4, \dots, (p_1, p_2, $	$(p_n)$ $(p_n)$ me) t	\\ ı\\ ı	removi removi .ight-lii	$\log p_2$ for $\log p_3$ for $\log p_3$ for $\log p_3$ for $\log p_3$	rom li rom li ance l	ist tak ist tak betwe	xes co xes co en tw	onstar onstar vo poi	nt tim nt tim ints p	e e and q
07 $08$ The function $d(p, q)$ 1. (5 points) Supp	$z = \text{Execute}($ $return \max(x)$ $q) \text{ returns (in the pose } T(n) \text{ is }$	$(p_1, p_2, p_4, \dots x, y, z)$ n constant ti	$(p_n)$ me) t	\\ ı he stra	removi .ight-lii	$\log p_3$ for the distance distance distance $p_3$	rom li ance l	ist tak	xes co en tw	onstar vo poi	nt tim	and q
The function $d(p, q)$ 1. (5 points) Supp	return $\max(x)$ q) returns (in cose $T(n)$ is	x, y,z) n constant ti	me) t	he stra	ight-lii	ne dista	ance l	betwee	en tw	vo poi	ints p	and q
The function $d(p, q)$ 1. (5 points) Supp	q) returns (in cose $T(n)$ is	n constant ti										
1. (5 points) Supp	pose $T(n)$ is											
2. (4 points) What	t is the heigh	nt of the recu	ırsion	tree fo	or $T(n)$	)?						
3. (3 points) How	many leaves	are in the re	ecursi	on tree	e for T	(n)?						

4. (3 points) What is the big-Theta running time of Execute?

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01 Wow(k.	n) \\ inputs are	e positive int	tegers	3												
02 if	f(n=1) return k															
03 e	else															
04	$half = \lfloor n/2 \rfloor$															
05	answer = Wo	pw(k,half) *	k Wo	w(k,ha	alf)											
06	if $(n \text{ is odd})$															
07		= answer*k														
08	return answe	r														
2. (3 points) V	What is the heigh	t of the recu	ırsion	tree fo	or $T(n)$	)? (As	$\operatorname{sume}$	that	n is a	a pow	er of	2.)				

4. (4 points) What is the big-Theta running time of Wow?

6

5

4

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01 Fabricate(a_1, \ldots, a_n; b_1, \ldots, b_n) \setminus \text{input is 2 lists of n integers, n is a power of 2}
02
              if (n = 1)
03
                      return a_1b_1
04
              else
                      p = \frac{n}{2}
05
                      \operatorname{rv} = \operatorname{Fabricate}(a_1, \ldots, a_p, b_1, \ldots, b_p)
06
                      rv = rv + Fabricate(a_1, \dots, a_p, b_{p+1}, \dots, b_n)
07
08
                      rv = rv + Fabricate(a_{p+1}, \dots, a_n, b_{p+1}, \dots, b_n)
                      rv = rv + Fabricate(a_{p+1}, \ldots, a_n, b_1, \ldots, b_p)
09
10
                      return rv
```

1. (5 points) Suppose that T(n) is the running time of Fabricate on an input array of length n. Give a recursive definition of T(n). Assume that dividing the list in half takes O(n) time.

2. (4 points) What is the height of the recursion tree for T(n), assuming n is a power of 2?

3. (3 points) What is the amount of work (aka sum of the values in the nodes) at level k of this tree?

4. (3 points) How many leaves are in the recursion tree for T(n)? (Simplify your answer.)