Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A \cap B) \subseteq f(A) \cap f(B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

Solution: This is true. Suppose that y is in $f(A \cap B)$. Then (by the definition of image), there is an x in $A \cap B$ such that $f(x) = y$. But since x is in $A \cap B$, $x \in A$ and $x \in B$. So then $f(x) = y$ must be in both $f(A)$ and $f(B)$.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ — always [√] sometimes [ ] never [ ]

Pascal's identity states that $\binom{n}{k}$ is equal to — $\binom{n-1}{k} + \binom{n-1}{k-1}$ [√] $\binom{n-1}{k} + \binom{n-1}{k+1}$ [ ] $\binom{n-1}{k} + \binom{n-2}{k}$ [ ]

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is — a rational [ ] a set of rationals [ ] undefined [√] one or more rationals [ ] a power set [ ]

Set B is a partition of a finite set A. Then $|A| = |B|$. — always [ ] sometimes [√] never [ ]

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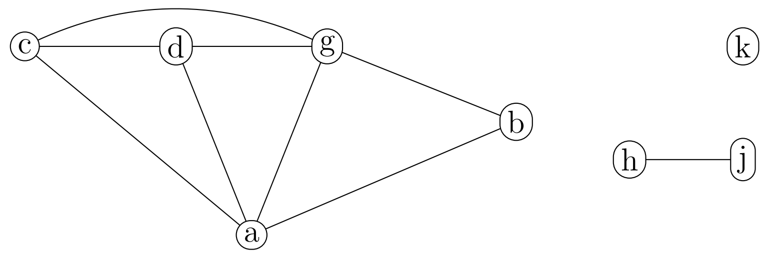
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$ **Solution:** 8

$f(d) =$ **Solution:** $\{cd, ad, dg\}$

$f(h) =$ **Solution:** $\{hj\}$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, T is not a partition of E. T contains all edges in E. However, $f(k)$ is the empty set, so T contains the empty set. Also, there is partial overlap between the subsets, e.g. $f(d)$ and $f(a)$ are different but share the edge ad.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(\{3\})$ is

an integer ☑	a set of integers ☐	undefined ☐
one or more integers ☐	a power set ☐	

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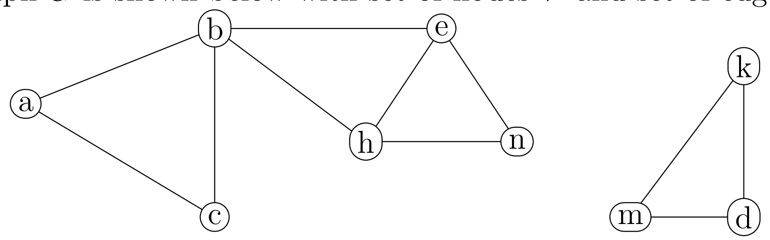
Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(k) =$

Solution: $\{m, d, k\}$

$F(b) =$

Solution: $\{a, b, c, e, n, h\}$

$|T| =$

Solution: 4

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is partial overlap between $F(c)$ and $F(h)$. But T doesn't contain the empty set and covers all of V.

(2 points) State Pascal's identity.

Solution:

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

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(7 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

Solution: For full credit, it's enough to give a well-explained specific example (e.g. using the relation $|a-b| \leq 10$) showing how partial overlap can occur.

Here's a more general (but still informal) argument. Since R is not transitive, there's three integers a, b, and c such that aRb and bRc but not aRc. Since aRb, $T(a)$ must contain a and b. Since bRc, $T(c)$ must contain c and b. So $T(a)$ and $T(c)$ overlap. But they can't be the same set because it's not true that aRc. So there is partial overlap.

Making the above argument full formal would require using the reflexive and symmetric properties of R.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is

an integer ☐ a set of integers ☐ undefined ☑ one or more integers ☐ a power set ☐

$\{\mathbb{N}\}$ is a partition of $\mathbb{N}$.

true ☑ false ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$

always ☐ sometimes ☑ never ☐

$\binom{n}{0}$

-1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

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(7 points) Suppose that $g : A \to B$ is an onto function. Let's define $F(y) = \{x \in A \mid g(x) = y\}$. Then define $P = \{F(y) \mid y \in B\}$. Is P a partition of A? Briefly justify your answer.

Solution: Yes, P is a partition of A. Notice that F produces all the pre-images of an output value y. So x is in $F(g(x))$ and x can't be in any other set produced by F. Because g is onto, any output value y has at least one pre-image. So $F(y)$ can never be empty.

(2 points) State the binomial theorem.

Solution:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☑ sometimes ☐ never ☐

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

$|\ \mathbb{P}(\{4,5,6,7,8\} \times \emptyset)\ |$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ 0 ☐ 1 ☑ 25 ☐ 2^5 ☐

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Solution: $\{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$:

Solution: $f(0, 36)$ is the line passing through the origin and $(0, 36)$.

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

Solution: Many possible answers here. For example, $\emptyset$, or any finite set or any circle.

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: This is not a partition of $\mathbb{R}^2$. It doesn't contain the empty set (good). And the elements of T do cover all of the plane (good). However, all the lines contain the origin, so there is partial overlap (bad).

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Q} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is

- a rational ☐
- a set of rationals ☑ (√)
- undefined ☐
- one or more rationals ☐
- a power set ☐