CS 173, Spring 18 — Examlet 12, Part B

Name: ______________________________

NetID: ____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A \cap B) \subseteq f(A) \cap f(B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ — always ☐ sometimes ☐ never ☐

Pascal's identity states that $\binom{n}{k}$ is equal to — $\binom{n-1}{k} + \binom{n-1}{k-1}$ ☐ $\binom{n-1}{k} + \binom{n-1}{k+1}$ ☐ $\binom{n-1}{k} + \binom{n-2}{k}$ ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is — a rational ☐ a set of rationals ☐ undefined ☐ one or more rationals ☐ a power set ☐

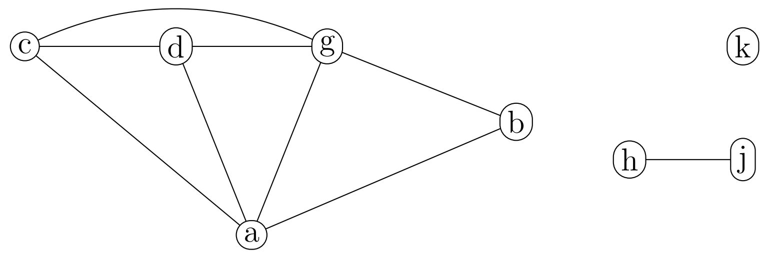
Set B is a partition of a finite set A. Then $|A| = |B|$. — always ☐ sometimes ☐ never ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(E)$ be defined by $f(n) = \{e \in E \mid n \text{ is an endpoint of } e\}$. And let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|V| =$

$f(d) =$

$f(h) =$

(7 points) Is T a partition of E? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(\{3\})$ is

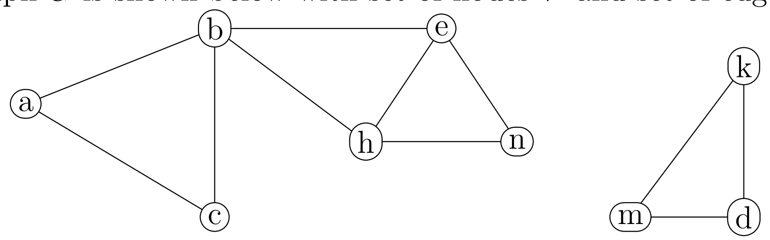
an integer ☐	a set of integers ☐	undefined ☐
one or more integers ☐	a power set ☐	

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$. Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(k) =$

$F(b) =$

$|T| =$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

(2 points) State Pascal's identity.

CS 173, Spring 18 — Examlet 12, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is — an integer ☐ a set of integers ☐ undefined ☐ one or more integers ☐ a power set ☐

$\{\mathbb{N}\}$ is a partition of $\mathbb{N}$. — true ☐ false ☐

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ — always ☐ sometimes ☐ never ☐

$\binom{n}{0}$ — -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Suppose that $g : A \to B$ is an onto function. Let's define $F(y) = \{x \in A \mid g(x) = y\}$. Then define $P = \{F(y) \mid y \in B\}$. Is P a partition of A? Briefly justify your answer.

(2 points) State the binomial theorem.

(6 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always ☐ sometimes ☐ never ☐

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐

$|\ \mathbb{P}(\{4,5,6,7,8\} \times \emptyset)\ |$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ 0 ☐ 1 ☐ 25 ☐ 2^5 ☐

CS 173, Spring 18 Examlet 12, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Describe (at a high level) the elements of $f(0, 36)$:

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Q} \to \mathbb{P}(\mathbb{Q})$ then $f(1.73)$ is

a rational ☐	a set of rationals ☐	undefined ☐
one or more rationals ☐	a power set ☐	