CS 173, Spring 18 Examlet 13, Part A

Name:____

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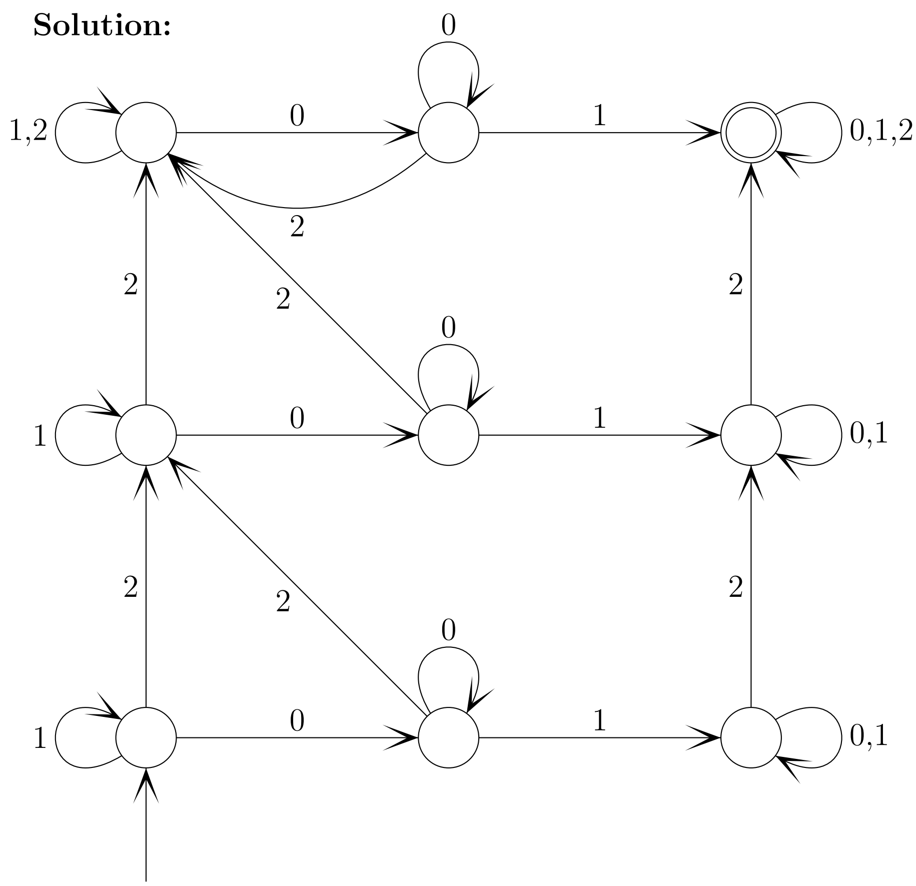
(15 points) Q needs your help designing an exploding keychain. To make it explode, James Bond must input 2 at least twice and also the sequence 01. These features can appear in any order, possibly separated. E.g. 0200112 will make it explode. The controller should read any sequence of the digits 0, 1, and 2. It should move into a final state as soon as it has seen the pattern, and remain in that final state as further characters come in. For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 8 states and, if you can, no more than 6.

The target number of states was wrong in the problem description. This requires 9 states.

Solution:



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(5 points) A black and white digitized picture is a (finite) 2D array of real values between 0 and 255. A color digitized picture consists of three such arrays (red, green, and blue). Is the set of all possible color digitzed pictures countable or uncountable?

Solution: It's uncountable. The real numbers between 0 and 255 are uncountable. So there are uncountably many options even for the first value in the red array.

(10 points) Check the (single) box that best characterizes each item.

Every mathematical function has a finite formula.	true ☐	false ☑ √	not known ☐
The rational numbers	finite ☐	countably infinite ☑ √	uncountable ☐
If $\mathbb{P}(A)$ is countable, then is A countable?	always ☑ √	sometimes ☐	never ☐
The set of all polynomials with rational coefficients.	finite ☐	countably infinite ☑ √	uncountable ☐
The set of all functions f from the even integers to the even integers	finite ☐	countably infinite ☐	uncountable ☑ √

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(5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Solution: This relation is not transitive. We have $(0, 0)T(-10, 10)$ (look at the second coordinate). We also have $(-10, 10)T(-5, -5)$ (look at the first coordinate). But it's not the case that $(0, 0)T(-5, -5)$.

(10 points) Check the (single) box that best characterizes each item.

For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$.

true [√] false [ ]

Suppose I want to estimate $\frac{103}{20}$. 3 is _____

an upper bound [ ] an exact answer [ ]
a lower bound [√] not a bound on [ ]

$\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$. (π is the familiar constant.)

true [√] false [ ] undefined [ ]

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = 7 - \lfloor \frac{x}{3} \rfloor$

onto [√] not onto [ ] not a function [ ]

$A \cap B \subseteq A$

true for all sets A and B [√] true for some sets A and B [ ]
false for all sets A and B [ ]

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(5 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n^2$. Give a recursive definition of f

Solution:

$f(0) = 0$, and $f(n+1) = f(n) + 2n + 1$ for $n \geq 0$.

You could also have used $f(n) = f(n-1) + 2n - 1$ for $n \geq 1$.

(10 points) Check the (single) box that best characterizes each item.

Suppose $f(n) \ll g(n)$.
Is $g(n) \ll f(n)$?

no [√] perhaps [ ] yes [ ]

V is the vertex set of a tree with n edges. $|\mathbb{P}(V)| =$

2^{n-1} [ ] 2^n [ ] not determined [ ]
2^{n+1} [√] n [ ]

The running time of Karatsuba's algorithm is recursively defined by $T(1) = d$ and $T(n) =$

$2T(n/2) + cn$ [ ] $3T(n/2) + cn$ [√]
$4T(n/2) + cn$ [ ] $4T(n/2) + c$ [ ]

The number of nodes in a full complete binary tree of height h

$\geq 2^h$ [ ] $2^{h+1} - 1$ [√]
$\leq 2^{h+1} - 1$ [ ] $\geq 2^{h+1} - 1$ [ ]

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is

an integer [ ] a set of integers [√] undefined [ ]
one or more integers [ ] a power set [ ]