CS 173, Spring 18 Examlet 13, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

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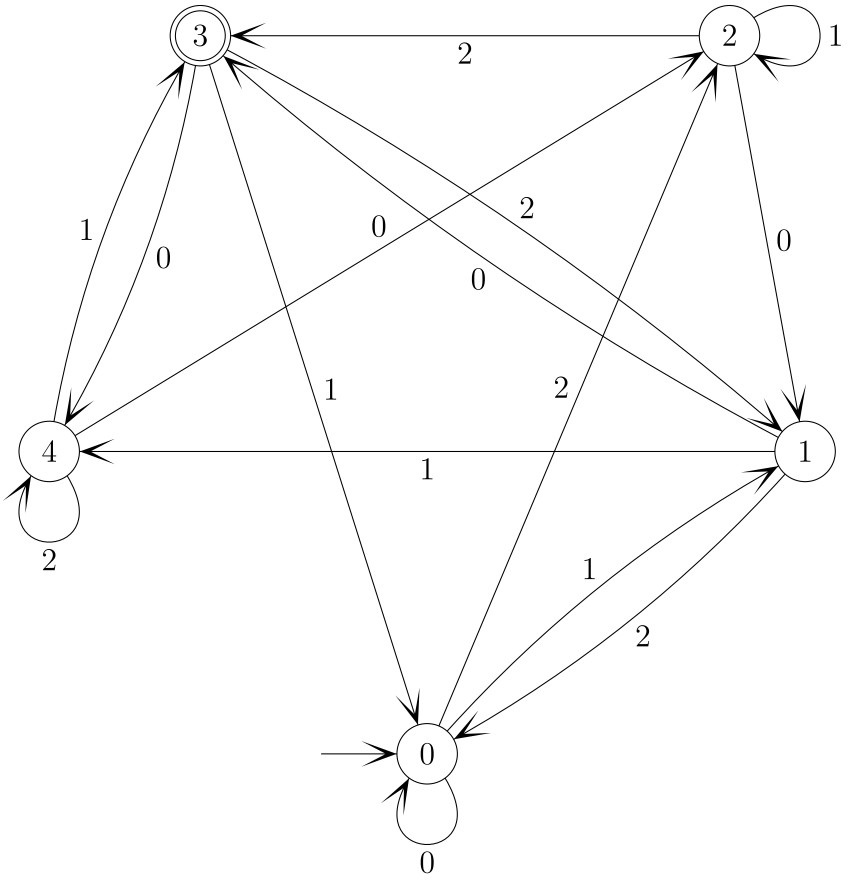
(15 points) Professor Martinez needs a state machine that will recognize certain base-3 numbers. It should read the digits in left-to-right order. That is, if you've seen number x and read a new digit d, your new number is $3x + d$. The machine should be in a final state whenever the number read so far is congruent to 3 (mod 5). For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a state diagram that will meet his needs, using no more than 7 states and, if you can, no more than 5.

Solution:



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(5 points) Let's consider two triangles in the real plane to be distinct if they have different vertices (so size and position matter). Also assume that all vertices have integer x and y coordinates. Is the set of distinct triangles countable or uncountable? Briefly justify your answer.

Solution: This set is countable. A triangle can be specified by three x-y pairs, i.e. six integers. We know that $\mathbb{Z}^k$ is countable for any positive integer k, so $\mathbb{Z}^6$ is countable.

(10 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\mathbb{Q})$ finite ☐ countably infinite ☐ uncountable ☑

There is a bijection $f : A \to B$ if and only if $|A| \leq |B|$. true ☐ false ☑ true for finite sets ☐

The set of all (finite) phone lattices using the 26 letters A, ..., Z. finite ☐ countably infinite ☑ uncountable ☐

The set of 10-digit US phone numbers. finite ☑ countably infinite ☐ uncountable ☐

Any function from $\{0, 1\}$ to $\mathbb{N}$ has a corresponding C++ program that computes it. true ☑ false ☐ not known ☐

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(5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.

$B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even integers starting with 2.

Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

Solution: $f(n) = 2(\sqrt{n} + 1)$

(10 points) Check the (single) box that best characterizes each item.

$\neg(p \to q) \equiv \neg p \to \neg q$ true ☐ false ☑

$\emptyset \times \emptyset =$ $\emptyset$ ☑ $\{\emptyset\}$ ☐ $\{\emptyset, \emptyset\}$ ☐ $\{(\emptyset, \emptyset)\}$ ☐

$29 \equiv 2 \pmod 9$ true ☑ false ☐

If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☑

Chromatic number of W_n. 2 ☐ 3 ☐ ≤ 3 ☐ ≤ 4 ☑

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(5 points) Suppose that w, x, y, and z are **positive** integers. How many solutions are there for the equation $w + x + y + z = 120$? Briefly explain or show work.

Solution: Since w, x, y, and z cannot be zero, we can rewrite our equation as $w' + x' + y' + z' = 116$, where $w' = w - 1$, $x' = x - 1$, $y' = y - 1$, and $z' = z - 1$. We can then view this as a problem of combinations with repetition, where we have four types (w, x, y, and z) and 116 objects. So the number of solutions is

$$\binom{119}{3} = \binom{119}{116}$$

(10 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$?

no ☐ perhaps ☐ yes ☑

All ways to assign
True/False values to
n input variables

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐ $\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☑

$T(1) = d$
$T(n) = 2T(n/2) + n$

$\Theta(n)$ ☐ $\Theta(n \log n)$ ☑ $\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐
$\Theta(n^{\log_3 2})$ ☐ $\Theta(n^{\log_2 3})$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

The root node of a tree is an
internal node

always ☐ sometimes ☑ never ☐

$\binom{n}{0}$

-1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐