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(15 points) Professor Martinez needs a state machine that will recognize certain base-3 numbers. It should read the digits in left-to-right order. That is, if you've seen number x and read a new digit d, your new number is $3x + d$. The machine should be in a final state whenever the number read so far is congruent to 3 (mod 5). For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a state diagram that will meet his needs, using no more than 7 states and, if you can, no more than 5.

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(5 points) Let's consider two triangles in the real plane to be distinct if they have different vertices (so size and position matter). Also assume that all vertices have integer x and y coordinates. Is the set of distinct triangles countable or uncountable? Briefly justify your answer.

(10 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\mathbb{Q})$	finite ☐	countably infinite ☐	uncountable ☐
There is a bijection $f : A \to B$ if and only if $\lvert A\rvert \leq \lvert B\rvert$.	true ☐	false ☐	true for finite sets ☐
The set of all (finite) phone lattices using the 26 letters A, ..., Z.	finite ☐	countably infinite ☐	uncountable ☐
The set of 10-digit US phone numbers.	finite ☐	countably infinite ☐	uncountable ☐
Any function from $\{0, 1\}$ to $\mathbb{N}$ has a corresponding C++ program that computes it.	true ☐	false ☐	not known ☐

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(5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.

$B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even integers starting with 2.

Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

(10 points) Check the (single) box that best characterizes each item.

$\neg(p \to q) \equiv \neg p \to \neg q$ true ☐ false ☐

$\emptyset \times \emptyset =$ $\emptyset$ ☐ $\{\emptyset\}$ ☐ $\{\emptyset, \emptyset\}$ ☐ $\{(\emptyset, \emptyset)\}$ ☐

$29 \equiv 2 \pmod 9$ true ☐ false ☐

If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☐

Chromatic number of W_n. 2 ☐ 3 ☐ ≤ 3 ☐ ≤ 4 ☐

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(5 points) Suppose that w, x, y, and z are **positive** integers. How many solutions are there for the equation $w + x + y + z = 120$? Briefly explain or show work.

(10 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $\Theta(g(n))$. Will $g(n)$ be $\Theta(f(n))$? no ☐ perhaps ☐ yes ☐

All ways to assign True/False values to n input variables

$\Theta(\log n)$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐	$\Theta(n^2)$ ☐
$\Theta(n^3)$ ☐	$\Theta(n^{\log_3 2})$ ☐	$\Theta(n^{\log_2 3})$ ☐	$\Theta(2^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + n$

$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐
$\Theta(n^{\log_3 2})$ ☐	$\Theta(n^{\log_2 3})$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

The root node of a tree is an internal node always ☐ sometimes ☐ never ☐

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☐ undefined ☐