CS 173, Summer 19 Examlet 2

Name:____

NetID:________________________ **Lecture: A**

Discussion: Monday & Wednesday 1:30 2:30

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers x and y, $x \neq 0$, if x and $\frac{y+1}{2}$ are rational, then $\frac{5}{x} + y$ is rational.

Solution: Let x and y be real numbers, where $x \neq 0$. Suppose that x and $\frac{y+1}{2}$ are rational.

By the definition of rational, $x = \frac{m}{n}$ and $\frac{y+1}{2} = \frac{p}{q}$, where m, n, p, and q are rationals, n and q non-zero. Since x is non-zero, m is also non-zero.

Since $x = \frac{m}{n}$ and x is not zero, $\frac{5}{x} = \frac{5n}{m}$.

Since $\frac{y+1}{2} = \frac{p}{q}$, $y + 1 = \frac{2p}{q}$. So $y = \frac{2p}{q} - 1 = \frac{2p-q}{q}$.

Combining these, we get that $\frac{5}{x} + y = \frac{5n}{m} + \frac{2p-q}{q} = \frac{5nq+2pm-qm}{mq}$. $5nq + 2pm - qm$ and mq are integers, since n, m, p, and q are integers. mq can't be zero because m and q are both non-zero. So $\frac{5}{x} + y$ is the ratio of two integers and therefore rational.

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod p$ and $p \mid q$, then $s \equiv t \pmod q$.

   Solution: This is false.

   Informally, since q is larger than p, congruence mod q makes finer distinctions among numbers than p does.

   More formally, consider $s = 1$, $t = 4$, $p = 3$ and $q = 6$. Then $3 \mid 6$ and s and t are congruent mod 3, but but s and t aren't congruent mod 6.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2262, 546)$. Show your work.

   Solution:

   $2262 - 546 \times 4 = 2262 - 2184 = 78$

   $546 - 7 \times 78 = 0$

   So the GCD is 78.

3. (4 points) Check the (single) box that best characterizes each item.

   $\gcd(p, q) = \dfrac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) always [√] sometimes [ ] never [ ]

   $7 \mid -7$ true [√] false [ ]