CS 173, Summer 19 Examlet 3

Name:____

NetID:________________________ **Lecture:** **A**

Discussion: **Monday & Wednesday** **1:30** **2:30**

$A = \{(x, y) \in \mathbb{Z}^2 \mid y = x^2 + 5x + 9\}$

$B = \{(a, b) \in \mathbb{Z}^2 \mid a \leq 2\}$

$C = \{(p, q) \in \mathbb{Z}^2 \mid q > 20\}$

Prove that $A \subseteq B \cup C$.

Solution: Let $(x, y) \in A$. By the definition of A, (x, y) is a pair of integers such that $y = x^2 + 5x + 9$. There are two cases:

Case 1: $x \leq 2$. Then $(x, y) \in B$, so $(x, y) \in B \cup C$.

Case 2: $x > 2$. The $y = x^2 + 5x + 9 > 4 + 10 + 9 = 21 > 20$. So $(x, y) \in C$, and therefore $(x, y) \in B \cup C$.

In both cases $(x, y) \in B \cup C$.

So any element of A is also an element of $B \cup C$, which means that $A \subseteq B \cup C$.

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1. (4 points) $A = \{\text{apple}, \text{maple}, \text{elm}, \emptyset\}$ $B = \{\text{tree}, \text{oak}, \emptyset\}$

   $A \cap B =$

   Solution: $\{\emptyset\}$

   $\{(p, q) \ : \ p \in \mathbb{Z}, \ q \in \mathbb{Z}, \text{ and } pq = 6\} =$

   Solution: $\{(p, q) \ : \ p \in \mathbb{Z}, \ q \in \mathbb{Z}, \text{ and } pq = 6\} =$
   $\{(1, 6), (6, 1), (2, 3), (3, 2)(-1, -6), (-6, -1), (-2, -3), (-3, -2)\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap B \subseteq A$ — true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

   $\forall x \in \mathbb{N}$, if $x^2 < -3$, then $x > 1000$. — true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{18} + [7]^4$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = [-3]^2 = [9]$

   $[7]^6 = ([7]^2)^3 = [-3]^3 = [-27] = [-1]$

   $[7]^{18} = ([7]^6)^3 = [-1]^3 = [-1] = [12]$

   So $[7]^{18} + [7]^4 = [12] + [9] = [21] = [8]$