CS 173, Summer 19 Examlet 4

Name:____

NetID:________________________ **Lecture:** **A**

Discussion: **Monday & Wednesday** **1:30** **2:30**

Recall how to multiply a real number α by a 2D point $(x, y) \in \mathbb{R}^2$: $\alpha(x, y) = (\alpha x, \alpha y)$.

Let $A = \mathbb{R}^+ \times \mathbb{R}^+$, i.e. pairs of positive real numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists a real number $\alpha \geq 1$ such that $(x, y) = \alpha(p, q)$.

Prove that $\gg$ is antisymmetric.

Solution: Let (x, y) and (p, q) be elements of A. Suppose that $(x, y) \gg (p, q)$ and $(p, q) \gg (x, y)$.

By the definition of $\gg$, there are real numbers $\alpha \geq 1$ and $\beta \geq 1$ such that $(x, y) = \alpha(p, q)$ and $(p, q) = \beta(x, y)$.

Substituting the second equation into the first, we get $(x, y) = \alpha\beta(x, y)$. This means that $\alpha\beta = 1$. Since $\alpha \geq 1$ and $\beta \geq 1$, this implies that $\alpha = \beta = 1$. So therefore $(x, y) = (p, q)$, which is what we needed to show.

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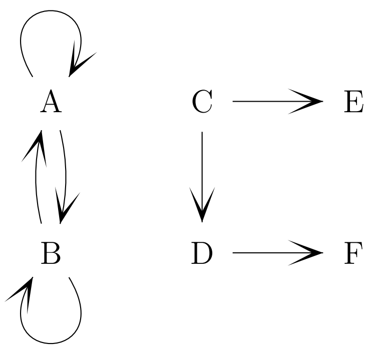
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

(That is, no boxes checked.)

2. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(a, b) \sim (p, q)$ if and only $aq = bp$. List three members of $[(5, 6)]$.

   Solution: (5,6), (10,12), (-5, -6)

3. (5 points) Let T be the relation on $\mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $(x, y) = \alpha(p, q)$ for some real number α. Is T symmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: T is not symmetric. We have $(0, 0)T(p, q)$ by setting α to zero but not $(3, 4)T(0, 0)$.