CS 173, Summer 19 Examlet 5

Name:____

NetID:____________________ **Lecture: A**

Discussion: Monday & Wednesday 1:30 2:30

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z} \to \mathbb{Z}^2$ by $g(n) = (|n|, f(n)|n|)$. Prove that g is one-to-one.

   Solution:

   Let p and q be integers. Suppose that $g(p) = g(q)$.

   By the definition of g, this means that $(|p|, f(p)|p|) = (|q|, f(q)|q|)$. So $|p| = |q|$ and $f(p)|p| = f(q)|q|$.

   Case 1: $|p| = 0$. Then $p = q = 0$. So $p = q$.

   Case 2: $|p|$ is non-zero. Substituting the first equation into the second, we get that $f(p)|p| = f(q)|p|$. So $f(p) = f(q)$. Since f is one-to-one, this means that $p = q$.

   So we've shown that $g(p) = g(q)$ implies that $p = q$, which means that g is one-to-one.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

   Solution: For every element y in C, there is an element x in M such that $g(x) = y$.

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1. (5 points) How many different 13-letter strings beginning with ma can be made be rearranging the letters in the word ``massachusetts''? Show your work.

   Solution: Notice that the first two letters are fixed. So there are 11 letters total to rearrange, with 4 copies of s, and two t's. So the number of possibilities is

$$\frac{11!}{4!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

The composition of two one-to-one functions is one-to-one.	true ☑	false ☐	
$f : \mathbb{N}^2 \to \mathbb{N}$ $f(p,q) = pq$	onto ☑	not onto ☐	not a function ☐
$g : \mathbb{R} \to \mathbb{Z}$ $g(x) = \|x\|$	one-to-one ☐	not one-to-one ☐	not a function ☑
Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.	true ☑	false ☐	
$\forall p \in \mathbb{Z}^+,\ \exists t \in \mathbb{Z}^+,\ \gcd(p,t) = 1$	true ☑	false ☐	