CS 173, Summer 19 Examlet 7

Name:____

NetID:________________________ **Lecture: A**

Discussion: Monday & Wednesday 1:30 2:30

(15 points) Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $7^n - 2^n$ is divisible by 5, for all natural numbers n.

Solution: Proof by induction on n.

Base case(s): At $n = 0$, $7^n - 2^n = 1 - 1 = 0$, which is divisible by 5.

Inductive hypothesis [Be specific, don't just refer to "the claim"]:

$7^n - 2^n$ is divisible by 5, $n = 0, 1, \ldots, k$ for some integer $k \geq 0$.

Rest of the inductive step:

Consider $7^{k+1} - 2^{k+1}$. We need to prove that it is divisible by 5.

$$\begin{aligned} 7^{k+1} - 2^{k+1} &= 7 \cdot 7^k - 2 \cdot 2^k = 2 \cdot 7^k + 5 \cdot 7^k - 2 \cdot 2^k \\ &= 2(7^k - 2^k) + 5 \cdot 7^k \end{aligned}$$

By the inductive hypothesis $7^k - 2^k$ is divisible by 5. So $2(7^k - 2^k)$ is divisible by 5. $5 \cdot 7^k$ is obviously divisible by 5.

$7^{k+1} - 2^{k+1}$ is the sum of two terms that are divisible by 5, so $7^{k+1} - 2^{k+1}$ is divisible by 5. This is what we needed to prove.

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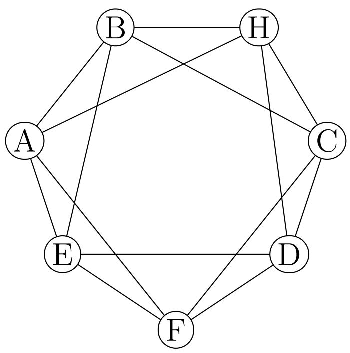
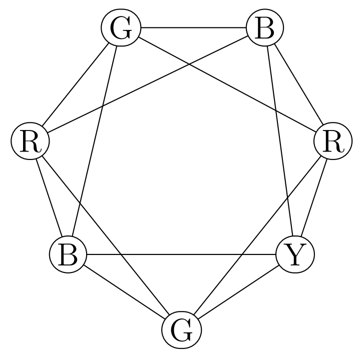
1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is 4. The picture above shows that the graph can be colored with four colors (upper bound).

To show the lower bound, let's try to color the graph with three colors. First color the triangle ABH as shown in the above picture. Then C must be colored R and E must be colored B. The colorings on A and E imply that F must be colored G. But none of the three colors is possible for D. So three colors isn't enough, i.e. we have a lower bound of 4.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of W_{2n}. 2 ☐ 3 ☑ ≤ 3 ☐ ≤ 4 ☐

$\sum_{i=1}^{p} i$ $\frac{p(p-1)}{2}$ ☐ $\frac{(p-1)^2}{2}$ ☐ $\frac{p(p+1)}{2}$ ☑ $\frac{(p-1)(p+1)}{2}$ ☐

Leal team's bridge collapsed under a 100 pound weight. 100 pounds is _____ on how much the bridge can hold.

an upper bound on ☑ exactly ☐

a lower bound on ☐ not a bound on ☐