CS 173, Summer 19 Examlet 9

Name:____

NetID:______________________ **Lecture: A**

Discussion: Monday & Wednesday 1:30 2:30

(18 points) A Borg tree is a full binary tree whose nodes contain integers such that

- Every leaf contains the value 0.
- The value $v(X)$ in a node X is (strictly) larger than the values in X's children.

Use (strong) induction to prove that the value in the root of a Borg tree is larger than the value in any other node of the tree.

Solution: The induction variable is named **h** and it is the **height** of/in the tree.

Base Case(s): $h = 0$. A tree of height zero contains only one node. It's value is (vacuously) larger than all the other nodes in the tree because there are no other nodes.

Inductive Hypothesis [Be specific, don't just refer to "the claim"]: Suppose that the root contains the largest value for all Borg trees of height $h = 0, 1, \ldots, k-1$

Inductive Step: Let T be a Borg tree of height $k > 0$. Since T is a full binary tree, its root r has two children p and q. Suppose that X is the subtree rooted at p and Y is the subtree rooted at q.

Both X and Y have height $< k$. Moreover, notice that X and Y are Borg trees because they are subtrees of T.

Suppose that x is any node of T, $x \neq r$. We need to show that $v(r) > v(x)$. There are three cases:

Case 1: x is the root of X or Y. Then $v(r) > v(x)$ by the definition of a Borg tree.

Case 2: x is any node other than p in the subtree X. Then $v(p) > v(x)$ by the inductive hypothesis, and $v(r) > v(p)$ by the definition of a Borg tree. So $v(r) > v(x)$.

Case 3: x is a node other than q in the subtree Y. Then $v(q) > v(x)$ by the inductive hypothesis, and $v(r) > v(q)$ by the definition of a Borg tree. So $v(r) > v(x)$. [It's ok to say this is just like Case 2.]

So, for any node x in T, $v(r) > v(x)$.

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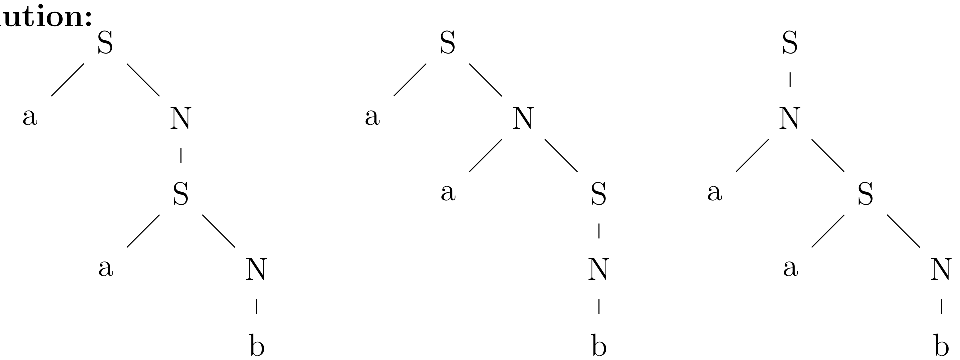
1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `aab` that match this grammar.

$$
\begin{aligned}
S &\rightarrow a\,N \mid N \\
N &\rightarrow a\,S \mid S \mid b
\end{aligned}
$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an n-node tree. Paths in opposite directions count as different.

n	☐	$2n$	☐	$\frac{n(n-1)}{2}$	☐
$n(n-1)$	☐	n^2	√	$\frac{n(n+1)}{2}$	☐

The level of the root node in a tree of height h.

-1	☐	0	√	1	☐	$h-1$	☐	h	☐