

Name: _____

NetID: _____ Lecture: A

Discussion: Monday & Wednesday 1:30 2:30

(18 points) A Borg tree is a full binary tree whose nodes contain integers such that

- Every leaf contains the value 0.
- The value $v(X)$ in a node X is (strictly) larger than the values in X 's children.

Use (strong) induction to prove that the value in the root of a Borg tree is larger than the value in any other node of the tree.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

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1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b . Draw three parse trees for the string **aab** that match this grammar.

$$\begin{aligned} S &\rightarrow a N \mid N \\ N &\rightarrow a S \mid S \mid b \end{aligned}$$

2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an n -node tree. Paths in opposite directions count as different.

n	<input type="checkbox"/>	$2n$	<input type="checkbox"/>	$\frac{n(n-1)}{2}$	<input type="checkbox"/>
$n(n-1)$	<input type="checkbox"/>	n^2	<input type="checkbox"/>	$\frac{n(n+1)}{2}$	<input type="checkbox"/>

The level of the root node in a tree of height h .

-1	<input type="checkbox"/>	0	<input type="checkbox"/>	1	<input type="checkbox"/>	$h-1$	<input type="checkbox"/>	h	<input type="checkbox"/>
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