CS 173, Summer 19 Examlet 13

Name:_____

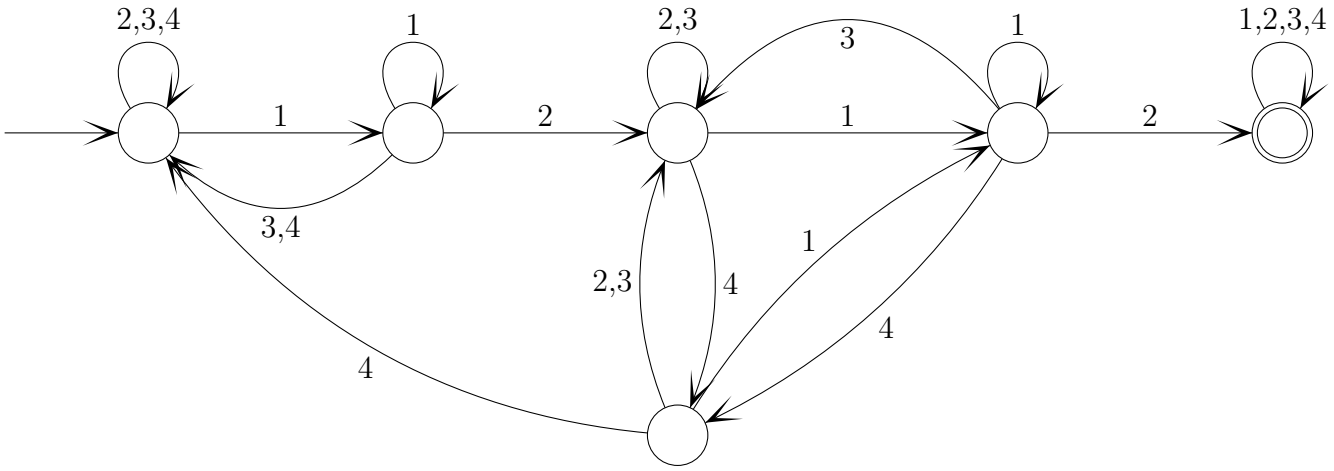
NetID:_____ **Lecture: A**

Discussion: Monday & Wednesday 1:30 2:30

(15 points) Q needs your help designing an exploding keychain. The keychain has four buttons, labelled 1, 2, 3, and 4. To make it explode, James Bond must input 12 twice. The two copies of 12 could be together (1212) or separated by other digits (1234312). Your state machine should move into an end state when that happens and remain in that final state as further digits come in. Exception: if you aren't already in the end state, two consecutive 4's (44) should abort the command (i.e. put the controller back in the start state). For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

Solution:



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(5 points) A "red/black tree" is a binary tree, each of whose nodes contains either "red" or "black." Is the set of all red/black trees countable or uncountable? Briefly justify your answer.

Solution: This is countable. For any n, there are only a finite number of distinct binary trees with n nodes. A tree with n nodes can be colored in 2^n ways. So there can only be a finite number of red/black trees of each size. Then the whole set is the union of countably many finite sets, which is countable.

(10 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(\mathbb{R})$ has the same cardinality as $\mathbb{R}$.	true	☐	false	☑	not known	☐
A product of countable sets is countable.	true	☐	false	☐	true for finite products	☑
The set of piano tunes, i.e. finite sequences of notes found on the standard piano keyboard.	finite	☐	countably infinite	☑	uncountable	☐
Every function from $\{1, 2, 3\}$ to the reals has a finite formula.	true	☐	false	☑	not known	☐
The set of chords (simultaneous combinations of notes) playable on an 88-key piano.	finite	☑	countably infinite	☐	uncountable	☐

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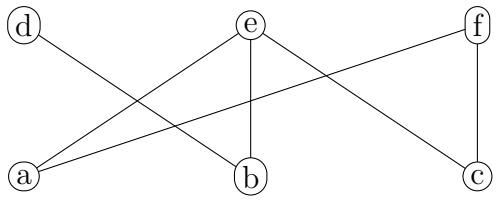
(5 points) Is this graph bipartite? Briefly justify your answer.



Solution: Yes, this is bipartite. All the edges go between the top set of nodes and the bottom set of nodes.

(10 points) Check the (single) box that best characterizes each item.

$p \wedge q \equiv \neg(p \to \neg q)$ true [√] false [ ]

If a and b are positive integers and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(r, a)$ true [ ] false [√]

$\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$ true [ ] false [√]

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = x + 2.137$ one-to-one [ ] not one-to-one [ ] not a function [√]

$\{1, 2\} \times \emptyset =$

$\emptyset$	[√]	$\{(1, \emptyset), (2, \emptyset)\}$	[ ]	$\{1, 2, \emptyset\}$	[ ]
$\{\emptyset\}$	[ ]	$\{1, 2\}$	[ ]	undefined	[ ]

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(5 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

Solution: There are positive reals c and k such that $0 \leq f(x) \leq cg(x)$ for every $x \geq k$.

(10 points) Check the (single) box that best characterizes each item.

Number of bit strings of length k.

2^k [√] $2^k - 1$ [ ] 2^{k-1} [ ] k [ ]

A tree with n nodes has _____ edges.

n [ ] $n-1$ [√] $\leq n$ [ ]

$n/2$ [ ] $\log n$ [ ]

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ [ ] $\Theta(\sqrt{n})$ [ ] $\Theta(n)$ [√] $\Theta(n \log n)$ [ ]

$\Theta(n^2)$ [ ] $\Theta(n^3)$ [ ] $\Theta(2^n)$ [ ] $\Theta(3^n)$ [ ]

$\binom{n}{1}$

-1 [ ] 0 [ ] 1 [ ] 2 [ ] n [√] undefined [ ]

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$

always [ ] sometimes [ ] never [√]