"CS/ECE 374 A": Algorithms & Models of Computation, Spring 2025

Midterm 1 — Feb 24, 2025

Name:	
NetID:	

- Please *clearly PRINT* your name and your NetID in the boxes above.
- This is a closed-book but you are allowed a 1 page (2 sides) hand written cheat sheet that you have to submit along with your exam. If you brought anything except your writing implements, put it away for the duration of the exam. In particular, you may not use *any* electronic devices.
- **Please read the entire exam before writing anything.** Please ask for clarification if any question is unclear. The exam has 6 problems, each worth 10 points.
- **You have 150 minutes (2.5 hours) for the exam.**
- If you run out of space for an answer, continue on the back of the page, or on the blank pages at the end of this booklet, **but please tell us where to look.**
- **Write everything inside the box around each page.** Anything written outside the box may be cut off by the scanner.
- **Proofs are required only if we specifically ask for them.** You may state and use (without proof or justification) any results proved in class or in the problem sets unless we explicitly ask you for one.
- You can do hard things!
- **Do not cheat.** You know the student code and all that jazz. Grades do matter, but not as much as you may think, and your values are more important.

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1 Regular Expression

Assume $\Sigma = \{0, 1\}$. Give regular expressions for the two languages below and **briefly explain how your expressions work**.

(a) (5 pts) All strings which contain the substring 01010 and which have an even number of blocks of 0's. Recall that a block (or a run) is a non-empty maximal substring of the same symbol.

(b) (5 pts) $\{1^{2n}w0^n \mid n \geq 2, w \in \{0, 1\}^*\}$

2 DFA Design

Give DFAs for the two languages below and **briefly describe the meaning of each state**.

(a) (5 pts) Strings in $\{0,1\}^*$ whose first character equals its second-to-last character. Examples of strings in this language include 00101, 11111, and 10. Examples of strings not in this language include ϵ, 1100, and 10101.

(b) (5 pts) Strings in $\{0,1,2\}^*$ that do *not* contain the substring 012.

3 NFA Design

(10 pts) Let $k \geq 1$ be an integer and let

$L_k = \{w \in \{0,1\}^* : |w| \geq k$ and number of 0s and 1s differ by at most two in the last k bits of $w\}$

For example if $k = 5$ then the strings 0000011 and 11010 are in L_5 while 11110 and 100000 are not in L_5. Describe an NFA for L_k with $O(k^2)$ states and **briefly describe the meaning of each state**.

4 Context-Free Grammars

Give a context-free grammar for the two languages below, where the terminal set T is $\{0, 1\}$. In order to get full credit you need to **briefly explain how your grammar works, and the role of each non-terminal**.

(a) (5pts) $L = \{0^i 1 0^j 1 0^k \mid i + k \geq 3j\}$

(b) (5 pts) All strings that have an odd number of 0's and end in 01.

5 Fooling Sets and Non-Regularity

(10 pts) Prove that the language

$$L = \{w \in \{0,1\}^* \mid w \text{ is a palindrome and } w \text{ has at least three 0s}\}$$

is not regular by providing a fooling set for it and showing that every pair of strings in your fooling set can be distinguished.

6 Language Transformation

(10 pts) Let $\Sigma = \{0, 1\}$. For a language $L \subseteq \Sigma^*$ we define an operation **deletepropermid** as follows:

$$\textbf{deletepropermid}(L) = \{uw \mid uvw \in L \text{ and } u, v, w \in \Sigma^*, |v| \geq 2\}.$$

Prove that if L is regular then **deletepropermid**(L) is also regular.

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