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ECE 120: Introduction to Computing

Fall 2024 – Midterm 1

Tuesday, September 24

- Be sure that your exam booklet has 14 pages.
- **Write your NAME and UIN clearly**
- Do not tear the exam booklet apart. You can only detach the last page for scratch work if needed.
- This is a closed book/notes exam. You may use a calculator.
- You are allowed one *handwritten* sheet of notes (both sides). Notes must be in English!
- Absolutely no interaction between students is allowed.
- Clearly indicate any assumptions that you make.
- **The questions are not weighted equally**. Budget your time accordingly.
- Show your work.
- **Illegible handwriting will be graded as incorrect**
- You must put your **UIN (9-digit #)** on **each page**

NetID:	UIN:

Name: KEY

Problem 1	23 points	_____
Problem 2	29 points	_____
Problem 3	10 points	_____
Problem 4	20 points	_____
Problem 5	18 points	_____
Total	100 points	_____

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Problem 1 (23 points): Representations

1. Encode the **decimal value 33** into each of the following representations (use minimum number of required bits for parts a and b)

   a. **(2 points)** unsigned representation: 100001

   b. **(2 points)** 2's complement representation: 0100001

   c. **(2 points)** Hexadecimal: 21

2. Given the **bit pattern 0111 0100,** calculate the **decimal equivalent** of this bit pattern when it is interpreted in each of the following representations:
   (Note: Truncate the most significant bit if the representation has 7-bit)

   a. **(2 points) 7-bit** unsigned : 116 (decimal equivalent)

   b. **(2 points) 7-bit** 2's complement : -12 (decimal equivalent)

   c. (**2 points**) **8-bit** 2's complement : 116 (decimal equivalent)

Floating-point representation

3. In this problem, you can refer to the IEEE 754 single-precision format provided on the last page of this exam booklet. You will encode the decimal fraction **-11.71875** using IEEE 754 single-precision floating-point.

   a. **(3 points)** Begin by converting **11.71875** into binary.

   Answer: 1011.10111

   b. **(2 points)** Rewrite your answer to **part (a)** using normalized binary scientific notation, for example, 1.001011×2^{13}.

   Answer: **1.** 01110111 × **2** 3

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c. (2 points) Fill in the bits below to represent -**11.71875** in IEEE 754 single-precision floating-point. Some bits have been filled in for you already.

FIRST GROUP OF 16 BITS

1	1	0	0	0	0	0	1	0	0	1	1	1	0	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

SECOND GROUP OF 16 BITS

4. (4 points) The numbers A and B below are represented in IEEE 754 single-precision floating-point.

FIRST GROUP OF 16 BITS for A

1	0	1	1	1	1	1	1	0	0	0	1	0	1	0	1

0	0	1	1	1	1	0	1	0	1	1	1	0	1	1	0

SECOND GROUP OF 16 BITS for A

FIRST GROUP OF 16 BITS for B

1	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1

0	0	1	1	1	1	0	1	0	1	1	1	0	1	1	0

SECOND GROUP OF 16 BITS for B

An arithmetic operation was applied on A to obtain B. In other words,

$$A \boxed{\text{*2^44}} = B$$

Fill in the blank by writing an arithmetic operator and a number, e.g., $\boxed{+1}$

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Problem 2 (29 points): Binary Arithmetic

1. Perform the following bit extension for the bit pattern 101011. The number represented should not change.

   a. **(1 point)** From 6-bit unsigned to 8-bit unsigned 00101011

   b. **(1 point)** From 6-bit 2's complement to 8-bit 2's complement: 11101011

   c. **(1 point)** How do you extend bit patterns in general for 2's complement representation? Explain using no more than 15 words.
   To extend an n-bit 2's complement to (n+k)-bit representation, we need to append the sign-bit k times to the left position.

2. a. **(0.5 points)** What is the bit pattern for the smallest number in 4-bit unsigned representation?
   0000

   b. **(0.5 points)** What is this smallest number from 2(a) in decimal value?
   0

   c. **(0.5 points)** What is the bit pattern for the largest number in 4-bit unsigned representation?
   1111

   d. **(0.5 points)** What is this largest number from 2(c) in decimal value?
   15

   e. **(1 point)** What is the numerical range of a *k*-bit unsigned representation? Express your answer as a function of *k* in the form of [*min*, *max*].

   $[\ 0\ ,\ 2^k-1\ ]$

3. a. **(0.5 points)** What is the bit pattern for the smallest number in 4-bit 2's complement representation? (Note: -1 > -2, meaning -2 is smaller than -1.)
   1000

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b. **(0.5 points)** What is this smallest number from 3(a) in decimal value?

-8

c. **(0.5 points)** What is the bit pattern for the largest number in 4-bit 2's complement representation?

0111

d. **(0.5 points)** What is this largest number from 3(c) in decimal value?

7

e. **(1 point)** What is the numerical range of a *k*-bit 2's complement representation? Express your answer as a function of *k* in the form of [*min*, *max*].

$[\ -2^{k-1}\ ,\ 2^{k-1}-1\ ]$

4. a. **(0.5 points)** For an odd non-negative integer, the rightmost bit in its unsigned representation is always:

"1"

b. **(0.5 points)** For an even negative integer, the leftmost bit in its 2's complement representation is always:

"1"

5. Perform the following 8-bit binary arithmetic operations. Indicate whether each operation results in an overflow when the bit patterns are interpreted using the indicated representation by circling the corresponding YES or NO. Show your work.

Let G **= xE6,** H **= xA9, P** = 01001011 **and Q** = 01010111

G = 1 1 1 0 0 1 1 0
H = 1 0 1 0 1 0 0 1

a. **(2 points) G + H =** 1 1 0 0 0 1 1 1 1

b. **(1 point)** Does overflow occur in (a) when the numbers have an unsigned representation?

Yes **No**

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c. **(1 point)** Explain your reasoning for (b) in no more than 10 words.

The addition operation results in a - carry out from the MSB position

d. **(1 point)** Does overflow occur in (a) when the numbers have a 2's complement representation?

Yes **(No)**

e. **(1 point)** Explain your reasoning for (d) in no more than 10 words.

The sign bits of the addend and the result are the same that is why no overflow.

f. **(2 points)** What is P +Q in 2's complement representation?

P = 0 1 0 0 1 0 1 1
Q = 0 1 0 1 0 1 1 1
P + Q = 1 0 1 0 0 0 1 0

g. **(1 point)** Does overflow occur in (f)? **(Yes)** **No**

h. **(1 point)** Explain your reasoning for (g) in no more than 10 words.

The addition of two positive numbers results in a negative number. [Sign bits are different]

6. Let **W =** 10111100 and **Z =** 1001

a. **(2 points)** Compute **W - Z** using 8-bit **2's complement representation for W, Z and the result**. (Show your work.)

W = 1 0 1 1 1 1 0 0
-Z = 0 0 0 0 0 1 1 1
W - Z = 1 1 0 0 0 0 1 1

Z = 1 1 1 1 1 0 0 1
-Z = 0 0 0 0 0 1 1 1

b. **(1 point)** Does overflow occur in (a)? **Yes** **(No)**

c. **(1 point)** Explain your reasoning for (b) in no more than 10 words.

Adding a positive number to a negartive number will always produce no overflow

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5. (2 points) Binary, Characters, and ASCII

Using the attached ASCII table write the sequence of 7-bit ASCII codes that corresponds to the string **`Hey!`**

Hex sequence 48 65 79 21

Binary sequence 1001000 1100101 11111001 0100001

6. (3 points) In a C program, suppose we declare: **`int A = -2`**. What do the following expressions evaluate to? Write the answer in **the requested** form and show your work. (Assume int is a 32-bit 2's complement integer.)

a. A >> 4 Answer in decimal: -1

b. 1 << 5 Answer in hex: x00000020

c. A | (~(1<<3)) Answer in hex: xFFFFFFFF

Problem 3 (10 points): Boolean Functions and Truth Table

(**4 points**) Calculate the outputs of the following two Boolean functions of three variables:

$$S(A, B, C) = (A'B'C) + (A'BC')+(AB'C')+(ABC)$$

$$P(A, B, C) = (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C)$$

C	B	A	S	P
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

1. (**2 points**) What conclusion you can draw about the functions S and P from the completed truth table? Answer in less that 10 words.

S and P are equivalent SOP and POS expressions of the same function.

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2. (**4 points**) Perform the following logical operations. Express your answers in hexadecimal notation.

   a. NOT((NOT(xDEFA)) AND (NOT(xFFFF))) = xFFFF

   b. x00FF XOR x325C = x32A3

   show your work:

NOT(xFFFF) = 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
NOT(xDEFA) = 0 0 1 0 0 0 0 1 0 0 0 0 0 1 0 1
AND => 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
NOT => xFFFF

x00FF = 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
x325C = 0 0 1 1 0 0 1 0 0 1 0 1 1 1 0 0
ADD => 0 0 1 1 0 0 1 0 1 0 1 0 0 0 1 1
= x32A3

Problem 4 (20 points): C Program Analysis

A naïve programmer helped us with this problem but forgot to write comments in the code to help you understand the purpose of the program. Alas, if only this naïve programmer would have paid attention to our advice!

Program

```
#include <stdio.h>

int main()
{
    char choice;
    int n, i;
    int result = 0;

    printf("Enter choice (f for factorial, s for sum, p for
prime sum): \n"); /*Note the printf format is broken into two
lines for reading convenience */

    scanf("%c", &choice);
    printf("Enter a positive integer n: \n");
    if (1!=scanf("%d", &n))
        {
         printf("Invalid Input\n");
         return 3;
         }

    if (n <= 0)
    {
      printf("Welcome to ECE 120! n must be positive.\n");
      return 0;
    }

    if (choice == 'f')
    {
        result = 1;
        for (i = 1; i <= n; i++)
        {
            result *= i; //note: a*=i => a=a*i;
        }
        printf("Factorial of %d is %d\n", n, result);
    }
```

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```
    else if (choice == 's')
    {
result = n * (n + 1) / 2; // Sum of first n natural numbers

printf("Sum of first %d natural numbers is %d\n", n, result);
    }
    else if (choice == 'p')
    {
        int count = 0, num = 2;
        while (count < n)
        {
            int is_prime = 1;
            for (i = 2; i <= num / 2; i++)
            {
                if (num % i == 0)
                {
                    is_prime = 0;
                    break;
                }
            }
            if (is_prime)
            {
                result += num;
                count++;
            }
            num++;
        }

printf("Sum of first %d prime numbers is %d\n", n, result);
    }
    else
    {
        printf("Welcome to ECE 120! Invalid choice.\n");
    }

    return 0;
}
```

(The questions you need to answer can be found on the next page.)

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Given the C program on the previous page, write down the output EXACTLY as it will be printed on the screen when the program is executed and when the user chooses to type the following values:

1. What happens when the user inputs **f** as the **choice** and **4** as the integer **n**?

Answer:
Enter choice (f for factorial, s for sum, p for prime sum
Enter a positive integer n :
Factorial of 4 is 24

2. What will the program output if the user inputs **s as the choice** and **5** as the integer **n**?

Answer:
Enter choice (f for factorial, s for sum, p for prime sum
Enter a positive integer n :
Sum of first 5 natural number is 15

3. What will the program output if the user inputs **s as the choice** and **n** as the integer **n**?

Answer:
Enter choice (f for factorial, s for sum, p for prime sum
Enter a positive integer n :
Invalid Input

4. If the user enters **p** as the **choice** and **−2** as **n**, what will be the output?

Answer:
Enter choice (f for factorial, s for sum, p for prime sum
Enter a positive integer n :
Welcome to ECE 120! n must be positive.

5. What output does the program produce when the user inputs **x** as the **choice** and **3** as **n**?

Answer:
Enter choice (f for factorial, s for sum, p for prime sum
Enter a positive integer n :
Welcome to ECE 120! Invalid choice.

Problem 5 (18 points): C Program Completion

Complete the program below that computes Maclaurin series expansion for $\cot^{-1} x$:

$$\cot^{-1} x = \prod_{j=1}^{M} \frac{4j^2}{4j^2 - 1} - \sum_{i=0}^{N} (-1)^i \frac{x^{2i+1}}{2i+1} \quad \text{for } |x| < 1$$

Note that Π is the symbol for multiplication and Σ is the symbol for summation.

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Line	Program
01	#include <stdio.h>
02	int main()
03	{
04	int N, M, i, j;
05	float x, quotient;
06	float numerator_1, numerator_2 = 1;
07	float sum = 0.0, product = 1.0;
08	printf("Enter x, M and N: \n");
09	scanf("%f %d %d", &x, &M, &N);
10	
11	for (j = 1; j ≤ M; j=j+1)
12	{
13	/* Compute $4j^2$ */
14	numerator_1 = 4*j*j;
15	
16	/* Update product */
17	product = product * numerator_1/(numerator_1-1);
18	}
19	
20	for (i = 0; i < N+1; i=i+1)
21	{
22	if (i==0) { /* compute x^{2i+1} for i=0 */
23	numerator_2 = numerator_2 * x;
24	}
25	else { /* compute x^{2i+1} */
26	numerator_2 = numerator_2*x*x;
27	}
28	/* compute x^{2i+1} / (2i+1) */
29	quotient = numerator_2/(2*i+1);
30	if (i % 2 == 1){ /* $(-1)^i$ */
31	quotient = (-1) * quotient;
32	}
33	sum = sum + quotient;
34	}
35	printf("For x=%f,the result is %f \n", x, sum);
36	return 0;
37	}

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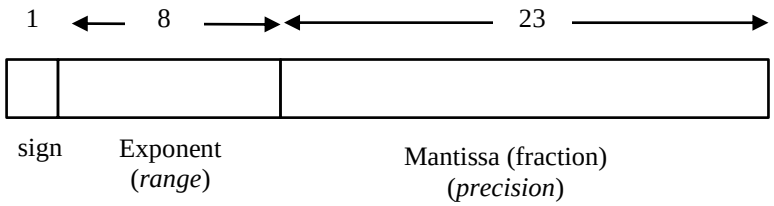
Table of ASCII Characters

Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex
(nul)	0	00	(sp)	32	20	@	64	40	`	96	60
(soh)	1	01	!	33	21	A	65	41	a	97	61
(stx)	2	02	"	34	22	B	66	42	b	98	62
(etx)	3	03	#	35	23	C	67	43	c	99	63
(eot)	4	04	$	36	24	D	68	44	d	100	64
(enq)	5	05	%	37	25	E	69	45	e	101	65
(ack)	6	06	&	38	26	F	70	46	f	102	66
(bel)	7	07	'	39	27	G	71	47	g	103	67
(bs)	8	08	(	40	28	H	72	48	h	104	68
(ht)	9	09	)	41	29	I	73	49	i	105	69
(lf)	10	0a	*	42	2a	J	74	4a	j	106	6a
(vt)	11	0b	+	43	2b	K	75	4b	k	107	6b
(ff)	12	0c	,	44	2c	L	76	4c	l	108	6c
(cr)	13	0d	-	45	2d	M	77	4d	m	109	6d
(so)	14	0e	.	46	2e	N	78	4e	n	110	6e
(si)	15	0f	/	47	2f	O	79	4f	o	111	6f
(dle)	16	10	0	48	30	P	80	50	p	112	70
(dc1)	17	11	1	49	31	Q	81	51	q	113	71
(dc2)	18	12	2	50	32	R	82	52	r	114	72
(dc3)	19	13	3	51	33	S	83	53	s	115	73
(dc4)	20	14	4	52	34	T	84	54	t	116	74
(nak)	21	15	5	53	35	U	85	55	u	117	75
(syn)	22	16	6	54	36	V	86	56	v	118	76
(etb)	23	17	7	55	37	W	87	57	w	119	77
(can)	24	18	8	56	38	X	88	58	x	120	78
(em)	25	19	9	57	39	Y	89	59	y	121	79
(sub)	26	1a	:	58	3a	Z	90	5a	z	122	7a
(esc)	27	1b	;	59	3b	[	91	5b	{	123	7b
(fs)	28	1c	<	60	3c	\	92	5c	|	124	7c
(gs)	29	1d	=	61	3d	]	93	5d	}	125	7d
(rs)	30	1e	>	62	3e	^	94	5e	~	126	7e
(us)	31	1f	?	63	3f	_	95	5f	(del)	127	7f

IEEE 754 32-bit floating point format



The actual number represented in this format is:

$$(-1)^{s} \times 1.\text{mantissa} \times 2^{\text{exp.} - 127}$$

where 1≤ exponent ≤ 254 for normalized representation.

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[Extra Page] – For Rough Sketching