

ECE 313: Exam II

Monday, April 8, 2024

7:00 p.m. — 8:15 p.m.

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Section: A, MWF at 10 am (Milenkovic) B, MWF at 11 am (Katselis)
 C, MWF at 1 pm (Shanbhag) D, MWF at 2pm (Bastopcu) CSP, Chicago (Shanbhag)

Instructions

WRITE YOUR NAME AND NETID ON TOP OF EVERY PAGE. This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of four problems worth a total of 100 points. The problems are not necessarily weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. You may use the back of the previous page as scratch paper, but we will only grade answers in the space provided. Draw a small box around each of your final numerical answers.

Grading	
1. 25 points	_____
2. 25 points	_____
3. 25 points	_____
4. 25 points	_____
Total (100 points)	_____

1. [25 points] A manufacturing company produces light bulbs and claims that the chance of a defective light bulb in their production line is 25%. However, a quality assurance team suspects that the chance of a defective light bulb is 50%. So, they independently sampled 3 light bulbs from the production line and noted the number of defective light bulbs.

(a) (8 points) Write out the likelihood matrix (table) for this problem. Find the ML decision rule using the likelihood matrix (table).

(b) (5 points) Find $p_{false\ alarm}$, p_{miss} , and p_e for the ML rule. To calculate p_e , assume that the prior probability of the company's claim being true is $1/3$.

(c) (7 points) Find the MAP rule, assuming that the prior probability of the company's claim being true is $1/3$.

(d) (5 points) Find $p_{false\ alarm}$, p_{miss} , and p_e for the MAP rule.

2. **[25 points]** A neuron fires signals according to a Poisson process with rate equal to $\lambda = 5$ signals/min. You are recording the neuronal activity.

(a) (5 points) Find the probability that you see no neuronal firing activity within the time interval $(3, 5]$ mins.

(b) (10 points) Find the probability that the neuron fired 2 times during the time interval $[1, 2]$ mins and 5 times during the interval $[1, 5]$ mins.

- (c) (10 points) A neuron is said to be active if it fires at least 3 times in the first 5 mins. If you recorded the neuron's activity only for the first 3 mins, and observed that it fired 2 times, what is the probability that the neuron is active?

3. [25 points] Suppose a *fair* die is flipped 180 times. Let the event A be defined as:

$$A = \{(\text{number of times 6 shows}) \leq (\text{number of times a number other than 6 shows}) + 4\}.$$

(a) (4 points) Let X be the random variable denoting the number of times a 6 shows. Express A in terms of X . What is the probability mass function of X ?

(b) (5 points) Write down an expression for the exact value of $P(A)$.

(c) (8 points) Using the Gaussian approximation with the continuity correction, approximate the value of $P(A)$ via the Φ function. Leave your answer in terms of the Φ function.

(d) (8 points) Let B denote the event $\{X = 55\}$. Using the Gaussian approximation with the continuity correction, approximate the value of $P(B)$ via the Q function. Leave your answer in terms of the Q function.

4. [25 points] Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{3}, & 0 \leq x \leq 3, \\ 1, & x > 3. \end{cases}$$

Let also $Y = X^2$.

(a) (16 points) Find $P(1 < X < 2)$ and $P(1 \leq X \leq \frac{3}{2})$.

(b) (9 points) Find $P(X \leq 3Y)$.